Midterm Exam I
CIS 341: Introduction to Logic and Automata — Spring 2001
Prof. Marvin K. Nakayama

Instructions:

- Write all of your answers in a Microsoft Word document, which you are to e-mail as an attachment to cis341DL@cis.njit.edu when you are done. Be sure to include your full name and student ID at the top of the document.
- The name of the Word file must be your last name followed by a dash and followed by your first name. For example, if your name is Joe Smith, then name your file Smith-Joe.doc .
- Right below your name and student ID, type in the following: "I have read and understand all of the instructions, and I will obey the Academic Honor Code. I will not discuss the exam with anyone other than possibly the course instructor." If you do not do this, then you will get a 0 on the exam.
- This exam has 4 pages in total, numbered 1 to 4. Make sure your exam has all the pages.
- This exam will be 2 hour and 30 minutes in length. You must e-mail as an attachment your Microsoft Word document containing your answers to cis341DL@cis.njit.edu by the end of the exam period. If you do not e-mail your solutions by the end of the exam period, you will receive a 0 on the exam.
- Send any questions you have during the exam to cis341DL@cis.njit.edu .
- This is an open-book, open-note exam.
- For all problems, follow these instructions:
  1. FA stands for finite automaton; TG stands for transition graph.
  2. For any proofs, be sure to provide a step-by-step argument, with justifications for every step. You may assume that the theorems in the textbook hold; i.e., you do not have to reprove the theorems in the textbook. When using a theorem from the textbook, make sure you refer to it by number (e.g., Theorem 3).

1. **[20 points]** For each of the following, specify TRUE if the statement is always correct. Otherwise, specify FALSE

(a) TRUE FALSE — A transition graph accepts Λ if and only if an initial state is also a final state.

(b) TRUE FALSE — If S^{**} is infinite, then S must be infinite.

(c) TRUE FALSE — If S^{*} is infinite, then S^{**} must be infinite.

(d) TRUE FALSE — If S^{*} is finite, then S must be infinite.

(e) TRUE FALSE — The regular expression $(a+\Lambda)(b+ba)^*a^*$ can generate the string $babb$.

(f) TRUE FALSE — If L is a non-regular language, then there is a non-deterministic finite automaton that accepts L.

(g) TRUE FALSE — A finite automaton may accept infinitely many different strings.

(h) TRUE FALSE — Every finite automaton is also a transition graph.

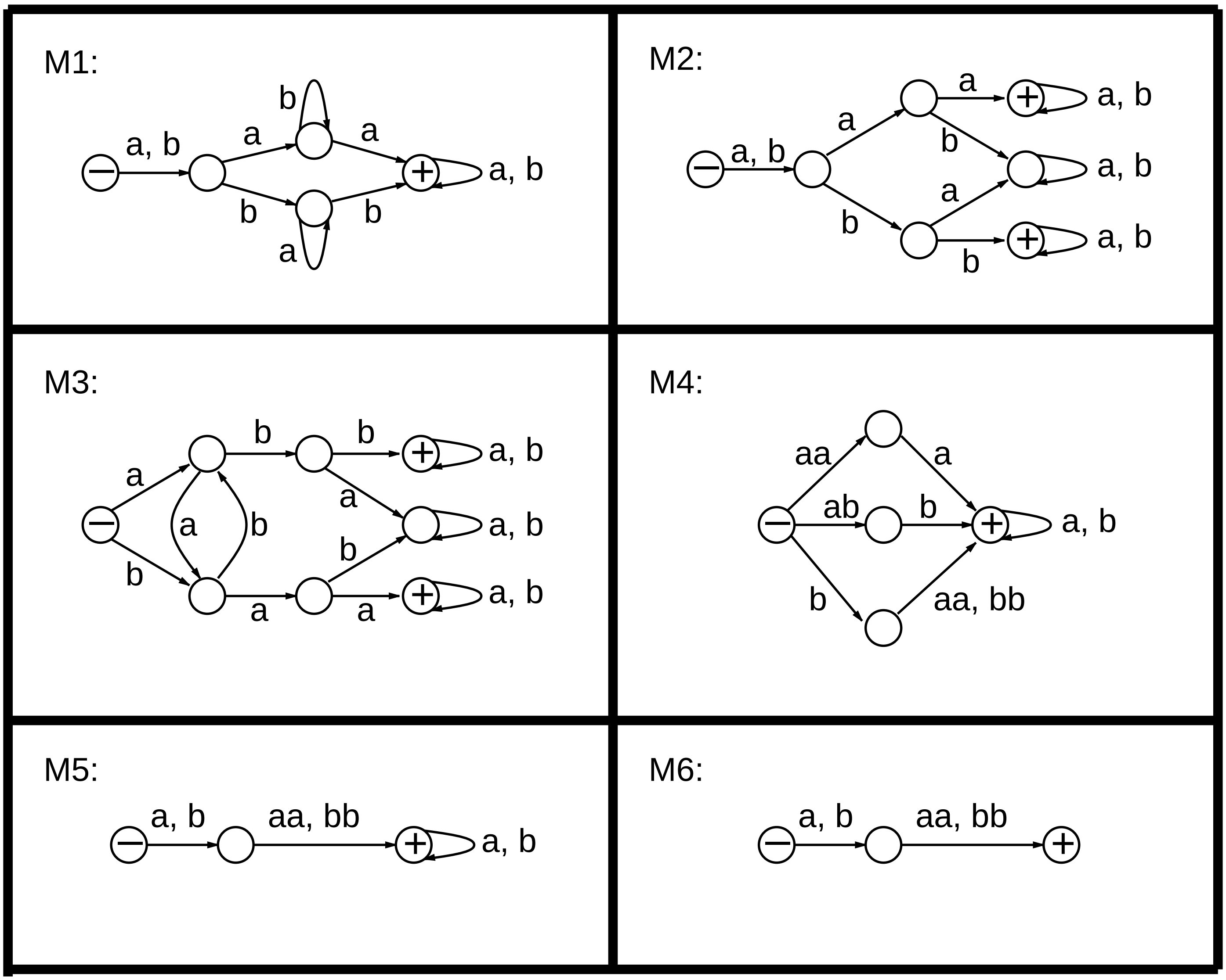
(i) TRUE FALSE — A non-deterministic finite automaton may accept infinitely many different strings.

(j) TRUE FALSE — A finite automaton can crash when processing a string.

2. **[30 points]** Let L be the language generated by the regular expression

$$(a+b)(aa+bb)(a+b)^*$$

with alphabet $\Sigma = \{a, b\}$. Consider the following 6 machines, labeled M1, M2, ... M6, where M1, M2, and M3 are finite automata, and M4, M5, and M6 are transition graphs:



For each machine above, indicate if it accepts exactly L or not. For each machine that does not accept exactly L, do the following:

- Show that it does not accept exactly L by giving either an example of a string $w \notin L$ that is accepted by the machine, or an example of a string $w \in L$ that is not accepted by the machine. Explain your example.
- Give a regular expression for the language exactly accepted by the machine.

3. **[20 points]** For each of the following languages L over the alphabet $\Sigma = \{a, b\}$, give a regular expression for L.

   (a) L exactly consists of all non-empty strings whose first letter is different from its last letter.

   (b) L exactly consists of all strings that do *not* contain the substring ab.

4. **[15 points]** Suppose that L is a non-regular language. Is it necessarily the case that L has infinitely many words?

   YES NO

   If your answer is YES, give a proof. If your answer is NO, give a counterexample. Explain your answer, and be sure to provide all details.

5. **[15 points]** Let L_1 and L_2 be languages, each having the same finite alphabet Σ. Suppose that

   - L_1 consists of all strings w such that $50 < \text{length}(w) < 100$.
   - L_2 is exactly accepted by some transition graph.

   Prove that $L_1^* + L_2$ is a regular language.