Midterm Exam I
CIS 341: Introduction to Logic and Automata — Spring 2002
Prof. Marvin K. Nakayama

Print Name (last name first): ____________________

Student Number: ____________________

I have read and understand all of the instructions below, and I will obey the Academic Honor Code.

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Signature and Date

- This exam has 6 pages in total, numbered 1 to 6. Make sure your exam has all the pages.
- This exam will be 1 hour and 25 minutes in length.
- This is a closed-book, closed-note exam.
- For all problems, follow these instructions:
  1. Give only your answers in the spaces provided. I will only grade what you put in the answer space, and I will take off points for any scratch work in the answer space. Use the scratch-work area to work out your answers before filling in the answer space.
  2. FA stands for finite automaton; TG stands for transition graph.
  3. For any proofs, be sure to provide a step-by-step argument, with justifications for every step.

Problem	1	2	3	4	5	Total
Points						

1. **[20 points]** For each of the following, circle TRUE if the statement is correct. Otherwise, circle FALSE

(a) TRUE FALSE — If L is any language, then L^* must be infinite.

(b) TRUE FALSE — If L is any language, then L^* must be finite.

(c) TRUE FALSE — If L_1 and L_2 are languages such that $L_1^* = L_2^*$, then $L_1 = L_2$.

(d) TRUE FALSE — A regular expression for the language $L = \{a^n : n = 1, 2, 3, \ldots\}$ is $\mathbf{a} + \mathbf{aa} + \mathbf{aaa} + \cdots$.

(e) TRUE FALSE — If L_1 is language having regular expression r_1 and L_2 is language having regular expression r_2, then the language $L_1 + L_2$ has regular expression $r_1 + r_2$.

(f) TRUE FALSE — Let L be the language over $\Sigma = \{a, b\}$ consisting of exactly all words that have either an even number of a's *or* an even number of b's. Then a regular expression for L is $(\mathbf{aa} + \mathbf{bb} + (\mathbf{ab} + \mathbf{ba})(\mathbf{aa} + \mathbf{bb})^*(\mathbf{ab} + \mathbf{ba}))^*$.

(g) TRUE FALSE — All transition graphs are non-deterministic.

(h) TRUE FALSE — If a transition graph accepts the string Λ, then some start state must also be a final state.

(i) TRUE FALSE — If a finite automaton accepts the string Λ, then the start state must also be a final state.

(j) TRUE FALSE — If $L_1 = \{\Lambda\}$ and $L_2 = \emptyset$, then $L_1 = L_2$.

2. **[20 points]** For each of the following languages L over the alphabet $\Sigma = \{a, b\}$, give a regular expression for L.

   (a) L exactly consists of all strings that begin with a and end with b.

   Regular Expression: ________________________________

   (b) L exactly consists of all strings that have an odd number of a's.

   Regular Expression: ________________________________

Scratch-work area

3. **[20 points]** For each of the following languages L over the alphabet $\Sigma = \{a, b\}$, give a finite automaton that accepts exactly L.

   (a) L exactly consists of all strings that have length of either 1 or 2.

   Draw finite automaton here:

   (b) L exactly consists of all strings that do not contain the substring aba.

   Draw finite automaton here:

Scratch-work area

4. **[20 points]** For each of the following parts, provide an example satisfying the given conditions. Give a brief explanation for each of your examples.

   (a) Give an example of a set S such that $S^* = S^+$.

   (b) Give an example of a set S such that $S = S^*$.

   (c) Give an example of a set S such that $S \neq S^+$.

   (d) Give an example of a set S such that S^* is finite.

5. **[20 points]** Suppose we define a restricted version of the C++ programming language in which variable names must satisfy all of the following conditions:

(i) A variable name can only use Roman letters (i.e., a, b, ..., z, A, B, ..., Z) or Arabic numerals (i.e., 0, 1, 2, ..., 9); i.e., underscore is not allowed.

(ii) A variable name must start with a Roman letter: a, b, ..., z, A, B, ..., Z

(iii) The length of a variable name must be no greater than 8.

(iv) A variable name cannot be a keyword (e.g., if). The set of keywords is finite.

Let L be the set of all valid variable names in our restricted version of C++.

(a) Let L_0 be the set of strings satisfying conditions (i), (ii), and (iii) above; i.e., we do not require condition (iv). How many strings are in L_0?

(b) Prove that L has a regular expression.