

Midterm Exam II

CIS 341: Introduction to Logic and Automata — Spring 2002, day sections

Prof. Marvin K. Nakayama

Print Name (last name first): \_\_\_\_\_

Student Number: \_\_\_\_\_

I have read and understand all of the instructions below, and I will obey the Academic Honor Code.

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Signature and Date

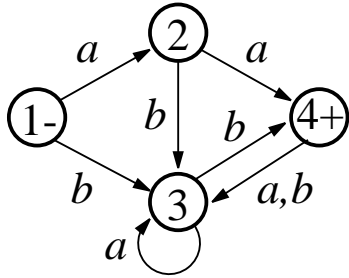
- This exam has 5 pages in total, numbered 1 to 5. Make sure your exam has all the pages.
- This exam will be 1 hour and 25 minutes in length.
- This is a closed-book, closed-note exam.
- For all problems, follow these instructions:
  1. Give only your answers in the spaces provided. I will only grade what you put in the answer space, and I will take off points for any scratch work in the answer space. Use the scratch-work area to work out your answers before filling in the answer space. If you need extra space for scratch work, use the back of your exam.
  2. FA stands for finite automaton; TG stands for transition graph.
  3. For any proofs, be sure to provide a step-by-step argument, with justifications for every step.

Problem	1	2	3	4	Total
Points					

1. [20 points] For each of the following, circle TRUE if the statement is correct. Otherwise, circle FALSE

- (a) TRUE FALSE — If  $L$  is a finite language, then  $L^*$  is a regular language.
- (b) TRUE FALSE — Regular languages are not closed under intersection.
- (c) TRUE FALSE — An effective procedure to determine if two finite automata  $F_1$  and  $F_2$  accept the same language is to process strings one at a time on both machines until finding one string that is accepted by one machine but not by the other.
- (d) TRUE FALSE — If  $L$  is a nonregular language, then  $L'$  is a nonregular language.
- (e) TRUE FALSE — Given a regular expression  $r$  generating a language  $L$ , there is an effective procedure to convert  $r$  into a finite automaton for  $L$ .
- (f) TRUE FALSE — Removing a finite number of words from a regular language results in a regular language.
- (g) TRUE FALSE — Suppose  $L$  is a language defined over an alphabet  $\Sigma$  and that  $L$  is accepted by some transition graph  $T$ . Then the set of all of the strings over  $\Sigma$  not accepted by  $T$  is a nonregular language.
- (h) TRUE FALSE — Every regular expression containing a Kleene star generates an infinite language.
- (i) TRUE FALSE — There is an effective procedure to determine if a transition graph accepts an infinite language.
- (j) TRUE FALSE — If a finite automaton  $F$  has at least one final state, then  $F$  accepts at least one string.

2. [40 points] Let  $L$  be the language over the alphabet  $\Sigma = \{a, b\}$  accepted by the following finite automaton:



- (a) Give a regular expression for  $L$ .

**Regular Expression:** \_\_\_\_\_

- (b) Give a finite automaton for the language  $L^*$ .  
Draw your finite automaton here:

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**Scratch-work area**

3. **[20 points]** For each of the following parts, provide an example satisfying the given conditions. Give a brief explanation for each of your examples.

(a) Give an example of languages  $L_1$  and  $L_2$  such that  $L_1 \subset L_2$ ,  $L_1$  is regular, and  $L_2$  is nonregular.

(b) Give an example of nonregular languages  $L_1$  and  $L_2$  such that  $L_1 + L_2$  is regular.

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Scratch-work area

4. [20 points] Recall the pumping lemma:

**Theorem 14** *Let  $L$  be a language accepted by a finite automaton having  $N$  states, and let  $w \in L$  with  $\text{length}(w) \geq N$ . Then there exists strings  $x$ ,  $y$ , and  $z$  such that*

- (i)  $w = xyz$
- (ii)  $y \neq \Lambda$
- (iii)  $\text{length}(x) + \text{length}(y) \leq N$
- (iv)  $xy^kz \in L$  for all  $k = 0, 1, 2, \dots$

Prove that the language  $L = \{b^{3m}a^{2m}b^{3m} : m = 0, 1, 2, \dots\}$  is nonregular.