Midterm Exam I
CIS 341: Introduction to Logic and Automata — Spring 2002, evening
Prof. Marvin K. Nakayama

Print Name (last name first): ______________________________

Student Number: ______________________________

I have read and understand all of the instructions below, and I will obey the Academic Honor Code.

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Signature and Date

- This exam has 8 pages in total, numbered 1 to 8. Make sure your exam has all the pages.
- This exam will be 3 hour and 5 minutes in length.
- This is a closed-book, closed-note exam.
- For all problems, follow these instructions:
  1. Give only your answers in the spaces provided. I will only grade what you put in the answer space, and I will take off points for any scratch work in the answer space. Use the scratch-work area to work out your answers before filling in the answer space.
  2. FA stands for finite automaton; TG stands for transition graph.
  3. For any proofs, be sure to provide a step-by-step argument, with justifications for every step.

Problem	1	2	3	4	5	6	7	Total
Points								

1. **[10 points]** For each of the following, circle TRUE if the statement is correct. Otherwise, circle FALSE

(a) TRUE FALSE — All languages are infinite.

(b) TRUE FALSE — All regular languages are infinite.

(c) TRUE FALSE — All nonregular languages are infinite.

(d) TRUE FALSE — All regular languages are finite.

(e) TRUE FALSE — If a transition graph accepts some language L, then there exists some nondeterministic finite automaton for the language L'.

(f) TRUE FALSE — If M is a finite automaton, then M is also a transition graph.

(g) TRUE FALSE — If L_1 is a regular language, then so is L_1^*.

(h) TRUE FALSE — If L is a nonregular language, then there exists a nondeterministic finite automaton that accepts L.

(i) TRUE FALSE — If L_1 and L_2 are languages such that $L_1 \subset L_2$ and L_2 is nonregular, then L_1 must be nonregular.

(j) TRUE FALSE — The regular expressions $(\mathbf{a} + \mathbf{b})^*$ and $(\mathbf{a}^*\mathbf{b}^*)$ generate the same language.

2. **[15 points]** For each of the following languages L over the alphabet $\Sigma = \{a, b\}$, give a regular expression for L.

   (a) L exactly consists of all strings containing exactly 2 a's.

   Regular Expression: ______________________________

   (b) L exactly consists of all strings with an even number of a's *or* an even number of b's. (Note that this says "*or*", not "*and*".)

   Regular Expression: ______________________________

Scratch-work area

3. **[15 points]** For each of the following languages L over the alphabet $\Sigma = \{a, b\}$, give a finite automaton that accepts exactly L.

   (a) L exactly consists of all strings whose first letter is a.

   Draw finite automaton here:

   (b) L exactly consists of all strings that do not end in bbb.

   Draw finite automaton here:

Scratch-work area

4. [**15 points**] Suppose that T is a transition graph with language L defined over an alphabet Σ. Suppose that a friend claims that you can create a transition graph T' for the language L' as follows:

   - T' has alphabet Σ.
   - The states and arcs in T' are the same as those in T, and the arcs have the same labels.
   - The start states of T' are the same as the start states of T.
   - Every final state in T is a non-final state in T', and every non-final state in T is a final state in T'.

   Show that your friend is wrong by giving an example of a transition graph T having a language L such that if we construct T' using the above rules, the language of T' is not L'. Be sure to explain your answer.

Scratch-work area

5. [**15 points**] For each of the following parts, provide an example satisfying the given conditions. Give a brief explanation for each of your examples.

   (a) Give an example of a set S such that $S = S^+$.

   (b) Give an example of sets S and T such that $ST = S$.

   (c) Give an example of sets S and T such that $ST = \emptyset$.

Scratch-work area

6. [**15 points**] Assume the following hold:

- Σ is an alphabet consisting of t letters.
- L_1 and L_2 are languages defined over Σ.
- There is a finite automaton FA_1 that accepts L_1, and FA_1 has m states.
- There is a finite automaton FA_2 that accepts L_2, and FA_2 has n states.

In class, we went over an algorithm to construct a finite automaton FA_3 for language L_1L_2 from FA_1 and FA_2. What is the maximum number of states in FA_3 constructed using this algorithm? Show your work.

Answer: ____

7. **[15 points]** Recall the pumping lemma:

Theorem 14 *Let L be a language accepted by a finite automaton having N states, and let $w \in L$ with $length(w) \geq N$. Then there exists strings x, y, and z such that*

(i) $w = xyz$

(ii) $y \neq \Lambda$

(iii) $length(x) + length(y) \leq N$

(iv) $xy^k z \in L$ *for all* $k = 0, 1, 2, \ldots$.

Prove that the language $L = \{b^m a b^m : m = 0, 1, 2, \ldots\}$ is nonregular.