Midterm Exam I
CIS 341: Introduction to Logic and Automata — Fall 2003, day
Prof. Marvin K. Nakayama

Print Name (family name first): ______________________________

I have read and understand all of the instructions below, and I will obey the Academic Honor Code.

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Signature and Date

- This exam has 5 pages in total, numbered 1 to 5. Make sure your exam has all the pages.
- This exam will be 1 hour and 25 minutes in length.
- This is a closed-book, closed-note exam.
- For all problems, follow these instructions:
  1. Give only your answers in the spaces provided. I will only grade what you put in the answer space, and I will take off points for any scratch work in the answer space. Use the scratch-work area to work out your answers before filling in the answer space.
  2. FA stands for finite automaton; TG stands for transition graph.
  3. For any proofs, be sure to provide a step-by-step argument, with justifications for every step.

Problem	1	2	3	4	Total
Points					

1. **[30 points]** For each of the following, circle TRUE if the statement is correct. Otherwise, circle FALSE

(a) TRUE FALSE — If $L = \{x^n : n \geq 0\}$, then a regular expression for L is $\Lambda + \mathbf{x} + \mathbf{xx} + \mathbf{xxx} + \cdots$.

(b) TRUE FALSE — If T is a transition graph in which none of the initial states is also a final state, then T cannot accept Λ.

(c) TRUE FALSE — A finite automaton can have more than one initial state.

(d) TRUE FALSE — All finite automata are deterministic.

(e) TRUE FALSE — All transition graphs are nondeterministic.

(f) TRUE FALSE — For any language L, there is exactly one finite automaton that accepts L.

(g) TRUE FALSE — If L is the language Palindrome over $\Sigma = \{a, b\}$, then L^* has regular expression $(\mathbf{a}^* + \mathbf{b}^*)^*$.

(h) TRUE FALSE — A finite automaton may crash while processing a string.

(i) TRUE FALSE — If a transition graph T accepts Λ, then the language of T is $\emptyset$.

(j) TRUE FALSE — For any language L, L^* is infinite.

2. **[25 points]** For each of the following languages L over the alphabet $\Sigma = \{0, 1\}$, give a regular expression for L.

   (a) L exactly consists of all non-empty strings whose first letter and last letter are not the same.

   Regular Expression: ______________________________

   (b) L exactly consists of all strings that have at most three 1's.

   Regular Expression: ______________________________

Scratch-work area

3. **[25 points]** For each of the following languages L over the alphabet $\Sigma = \{a, b\}$, give a finite automaton that accepts exactly L.

(a) L exactly consists of all strings with an odd number of a's and an even number of b's.

Draw finite automaton here:

(b) L exactly consists of all strings that have both ab and ba as substrings.

Draw finite automaton here:

Scratch-work area

4. **[20 points]** Let S be any set of strings. Prove that $S^* = S^+$ if and only if $\Lambda \in S$.