Midterm Exam II
CIS 341: Introduction to Logic and Automata — **Fall 2003, day**
Prof. Marvin K. Nakayama

Print family (or last) name: ____________________________________

Print given (or first) name: ____________________________________

I have read and understand all of the instructions below, and I will obey the Academic Honor Code.

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Signature and Date

- This exam has 5 pages in total, numbered 1 to 5. Make sure your exam has all the pages.
- This exam will be 1 hour and 25 minutes in length.
- This is a closed-book, closed-note exam.
- For all problems, follow these instructions:
  1. Give only your answers in the spaces provided. I will only grade what you put in the answer space, and I will take off points for any scratch work in the answer space. Use the scratch-work area or the backs of the sheets to work out your answers before filling in the answer space.
  2. FA stands for finite automaton; TG stands for transition graph.
  3. For any proofs, be sure to provide a step-by-step argument, with justifications for every step.

Problem	1	2	3	4	Total
Points					

1. **[30 points]** For each of the following, circle TRUE if the statement is correct. Otherwise, circle FALSE

(a) TRUE FALSE — If L is an infinite language, then L must be nonregular.

(b) TRUE FALSE — If L_1 and L_2 are nonregular languages then $L_1 \cap L_2$ must be nonregular.

(c) TRUE FALSE — If L_1 and L_2 are regular languages then $L_1 \cap L_2$ must be regular.

(d) TRUE FALSE — If language L is accepted by a nondeterministic finite automaton, then L must have a regular expression.

(e) TRUE FALSE — Suppose a language L over an alphabet Σ is accepted by a finite automaton. Then an effective procedure to decide if $L = \emptyset$ is to test every string in Σ^* to see if any are accepted by the FA.

(f) TRUE FALSE — There is an effective procedure to decide if a transition graph accepts an infinite language.

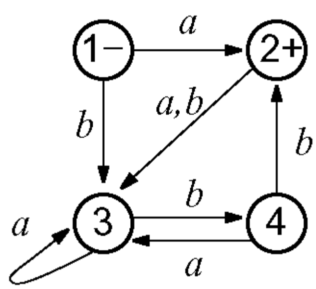
(g) TRUE FALSE — If L is the language Palindrome over $\Sigma = \{a, b\}$, then L^* has regular expression $(\mathbf{a} + \mathbf{b}^*)^*$.

(h) TRUE FALSE — If a language L is accepted by a transition graph, then there must exist a regular expression for L'.

(i) TRUE FALSE — If L is a nonregular language, then there is a nondeterministic finite automaton for L.

(j) TRUE FALSE — A regular expression for the language $L = \{a^m b^m : m \geq 0\}$ is $\Lambda + \mathbf{ab} + \mathbf{aa} + \mathbf{bb} + \mathbf{aaabbb} + \cdots$.

2. **[30 points]** For the language L of the finite automaton below, construct a finite automaton for L^*.



Draw FA for L^* here:

Scratch-work area

3. **[20 points]** Suppose that a language L_1 is accepted by a finite automaton $FA_1 = (K_1, \Sigma, \pi_1, s_1, F_1)$, and a language L_2 is accepted by a finite automaton $FA_2 = (K_2, \Sigma, \pi_2, s_2, F_2)$, where for FA_i, $i = 1, 2$, K_i is the set of states, Σ is the input alphabet, $\pi_i : K_i \times \Sigma \rightarrow K_i$ is the transition function, s_i is the initial state, and F_i is the set of final states. Define the language $L_3 = L_1 \cap L_2$, and for this question, you are to define a finite automaton $FA_3 = (K_3, \Sigma, \pi_3, s_3, F_3)$ for L_3 in terms of FA_1 and FA_2, as we covered in class.

   (a) Define the set K_3 of states of FA_3 in terms of FA_1 and FA_2.

   (b) Define the transition function π_3 of FA_3 in terms of FA_1 and FA_2.

   (c) Define the initial state s_3 of FA_3 in terms of FA_1 and FA_2.

   (d) Define the set F_3 of final states of FA_3 in terms of FA_1 and FA_2.

4. **[20 points]** Recall the pumping lemma:

Theorem 14 *Let L be a language accepted by a finite automaton having N states, and let $w \in L$ with length$(w) \geq N$. Then there exists strings x, y, and z such that*

(i) $w = xyz$,

(ii) $y \neq \Lambda$,

(iii) $\mathit{length}(x) + \mathit{length}(y) \leq N$,

(iv) $xy^k z \in L$ *for all* $k = 0, 1, 2, \ldots$.

For a string w over the alphabet $\Sigma = \{a, b\}$, define $n_a(w)$ to be the number of a's in w, and define $n_b(w)$ to be the number of b's in w. For example, if $w = abaa$, then $n_a(w) = 3$ and $n_b(w) = 1$. Prove that $L = \{w \in \Sigma^* : n_a(w) \geq n_b(w)\}$ is nonregular.