Midterm Exam
CIS 341-451: Introduction to Logic and Automata — Fall 2004, eLearning
Prof. Marvin K. Nakayama

Print Family (i.e., Last) Name: ____________________

Print Given (i.e., First) Name: ____________________

I have read and understand all of the instructions below, and I will obey the Academic Honor Code.

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Signature and Date

- This exam has 7 pages in total, numbered 1 to 7. Make sure your exam has all the pages.
- The exam is to be given on Saturday, October 23, 2004, 12:30–3:00pm.
- This is a closed-book, closed-note exam. No calculators are allowed.
- For all problems, follow these instructions:
  1. Give only your answers in the spaces provided. I will only grade what you put in the answer space, and I will take off points for any scratch work in the answer space. Use the scratch-work area or the backs of the sheets to work out your answers before filling in the answer space.
  2. FA stands for finite automaton; TG stands for transition graph.
  3. For any proofs, be sure to provide a step-by-step argument, with justifications for every step.

Problem	1	2	3	4	5	6	Total
Points							

1. **[20 points]** For each of the following, circle TRUE if the statement is correct. Otherwise, circle FALSE

(a) TRUE FALSE — If L is accepted by a transition graph, then there must be a regular expression for L.

(b) TRUE FALSE — All finite automata are also transition graphs.

(c) TRUE FALSE — A transition graph must have exactly one initial state.

(d) TRUE FALSE — The regular expressions $\mathbf{b}^*\mathbf{a}^*$ and $(\mathbf{ba})^*$ generate the same language.

(e) TRUE FALSE — If L is any language, then $\Lambda \in L^*$.

(f) TRUE FALSE — A finite automaton may have no final states.

(g) TRUE FALSE — L has a regular expression only if L is finite.

(h) TRUE FALSE — If L_1 is a regular language and L_2 is a nonregular language, then $L_1 + L_2$ must be a nonregular language.

(i) TRUE FALSE — If L_1 is a nonregular language, then there must exist a regular language L_2 such that $L_2 \subset L_1$.

(j) TRUE FALSE — There is an effective procedure to determine if two transition graphs accept the same language.

2. **[20 points]** For each of the following languages L over the alphabet $\Sigma = \{0, 1\}$, give a regular expression for L.

   (a) L consists of exactly those strings over Σ that begin and end with 1.

   Regular Expression: ______________________________

   (b) L consists of exactly those strings over Σ that have an odd number of 1's.

   Regular Expression: ______________________________

Scratch-work area

3. **[20 points]** For each of the following languages L over the alphabet $\Sigma = \{a, b\}$, give a finite automaton that accepts exactly L.

   (a) L exactly consists of all strings over Σ with length at least 2.

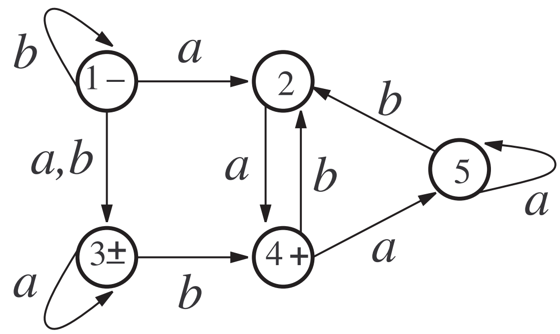
   Draw finite automaton here:

   (b) L exactly consists of all strings over Σ that end in aab.

   Draw finite automaton here:

Scratch-work area

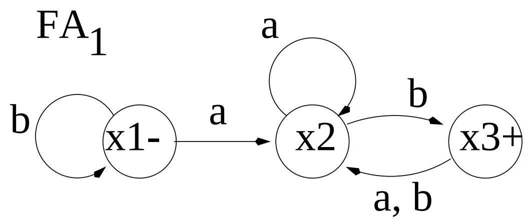
4. [**13 points**] Let L be the language accepted by the transition graph below. Give a regular expression for L.

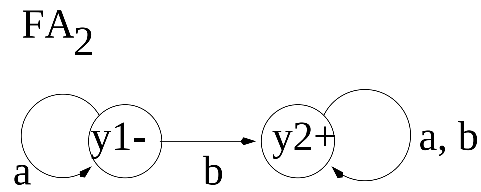


Regular Expression: ______________________________

Scratch-work area

5. [**12 points**] Let L_1 be the language accepted by the finite automaton FA_1 below, and L_2 be the language accepted by the finite automaton FA_2 below.





Draw a finite automaton for $L_1' \cap L_2$ here:

Scratch-work area

6. **[15 points]** Recall the pumping lemma:

Theorem 14 *Let L be a language accepted by a finite automaton having N states, and let $w \in L$ with length$(w) \geq N$. Then there exists strings x, y, and z such that*

(i) $w = xyz$,

(ii) $y \neq \Lambda$,

(iii) $\mathit{length}(x) + \mathit{length}(y) \leq N$,

(iv) $xy^kz \in L$ *for all* $k = 0, 1, 2, \ldots$.

Prove that $L = \{b^{3m}a^{2m}b^m : m \geq 0\}$ is a nonregular language.