Midterm Exam II
CIS 341: Introduction to Logic and Automata — **Fall 2004, day**
Prof. Marvin K. Nakayama

Print family (or last) name: ______________________________

Print given (or first) name: ______________________________

I have read and understand all of the instructions below, and I will obey the Academic Honor Code.

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Signature and Date

- This exam has 5 pages in total, numbered 1 to 5. Make sure your exam has all the pages.
- This exam will be 1 hour and 25 minutes in length.
- This is a closed-book, closed-note exam.
- For all problems, follow these instructions:
  1. Give only your answers in the spaces provided. I will only grade what you put in the answer space, and I will take off points for any scratch work in the answer space. Use the scratch-work area or the backs of the sheets to work out your answers before filling in the answer space.
  2. FA stands for finite automaton; TG stands for transition graph.
  3. For any proofs, be sure to provide a step-by-step argument, with justifications for every step.

Problem	1	2	3	4	Total
Points					

1. **[30 points]** For each of the following, circle TRUE if the statement is correct. Otherwise, circle FALSE

   (a) TRUE FALSE — Determining if a regular expression generates an infinite language is a decidable problem.

   (b) TRUE FALSE — If L is a regular language, then L' is regular.

   (c) TRUE FALSE — If L is a nonregular language, then L' is nonregular.

   (d) TRUE FALSE — If L_1 is a regular language and L_2 is another language such that $L_1 \subset L_2$, then L_2 must be regular.

   (e) TRUE FALSE — There is an effective procedure to determine if a transition graph accepts the empty language.

   (f) TRUE FALSE — If L_1 has a finite automaton and L_2 has a regular expression, then $(L_1L_2)'$ has a transition graph.

   (g) TRUE FALSE — The language $\{a^{2n} : n \geq 0\}$ is a nonregular language.

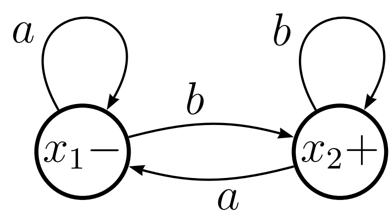
   (h) TRUE FALSE — If L has a context-free grammar, then L is a nonregular language.

   (i) TRUE FALSE — An effective procedures to determine if the language L of a transition graph T contains Λ is to conclude that $\Lambda \in L$ if and only if an initial state of T is also a final state.

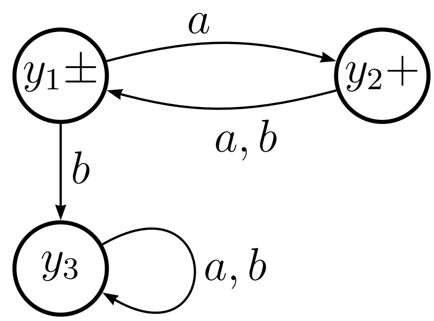
   (j) TRUE FALSE — Every regular language has a nondeterministic finite automaton.

2. **[30 points]** Let L_1 be the language of finite automata M_1 below, and let L_2 be the language of finite automata M_2 below.

FA M_1



FA M_2



Give a finite automaton for $L_1' \cap L_2$.

Scratch-work area

3. [**20 points**] Let $\Sigma = \{a, b\}$. Give context-free grammars for each of the languages below. Be sure to define the set of nonterminals, the set of terminals, and the productions.

   (a) $\{w \in \Sigma^* : w \text{ begins with } aa\}$.

   (b) EVEN-EVEN, which is the set of strings over Σ that have an even number of a's and an even number of b's.

   (c) $\{w \in \Sigma^* : w = w^R, |w| \text{ is even}\}$, where w^R is the reverse of the string w.

Scratch-work area

4. **[20 points]** Recall the pumping lemma:

Theorem 14 *Let L be a language accepted by a finite automaton having N states, and let $w \in L$ with length$(w) \geq N$. Then there exists strings x, y, and z such that*

(i) $w = xyz$,

(ii) $y \neq \Lambda$,

(iii) *length*(x) + *length*$(y) \leq N$,

(iv) $xy^kz \in L$ *for all* $k = 0, 1, 2, \ldots$.

Let $\Sigma = \{a, b\}$, and define language $L = \{w \in \Sigma^* : w = w^R, |w| \text{ is even}\}$, where w^R denotes the reverse of the string w. Prove that L is a nonregular language.