Midterm Exam I
CIS 341: Introduction to Logic and Automata — Spring 2004, **evening**
Prof. Marvin K. Nakayama

Print Family (i.e., Last) Name: ______________________________

Print Given (i.e., First) Name: ______________________________

I have read and understand all of the instructions below, and I will obey the Academic Honor Code.

____
Signature and Date

- This exam has 5 pages in total, numbered 1 to 5. Make sure your exam has all the pages.
- This exam will be 1 hour and 25 minutes in length.
- This is a closed-book, closed-note exam.
- For all problems, follow these instructions:
  1. Give only your answers in the spaces provided. I will only grade what you put in the answer space, and I will take off points for any scratch work in the answer space. Use the scratch-work area or the backs of the sheets to work out your answers before filling in the answer space.
  2. FA stands for finite automaton; TG stands for transition graph.
  3. For any proofs, be sure to provide a step-by-step argument, with justifications for every step.

Problem	1	2	3	4	Total
Points					

1. **[30 points]** For each of the following, circle TRUE if the statement is correct. Otherwise, circle FALSE

(a) TRUE FALSE — Suppose that T is a transition graph defined with an alphabet Σ. If every state in T is a final state, then T accepts Σ^*.

(b) TRUE FALSE — A transition graph may crash when processing a string.

(c) TRUE FALSE — If L is an infinite language, then $L = L^*$.

(d) TRUE FALSE — The regular expressions $\mathbf{a}^*\mathbf{b}^*$ and $(\mathbf{ab})^*$ generate the same language.

(e) TRUE FALSE — If $\Lambda \in L$, then $L^+ = L^*$.

(f) TRUE FALSE — For any language L, $\Lambda \in L^+$.

(g) TRUE FALSE — A transition graph may have no final states.

(h) TRUE FALSE — A finite automaton can have more than one final state.

(i) TRUE FALSE — If $L = \emptyset$, then $\Lambda \in L$.

(j) TRUE FALSE — A regular expression for the language $L = \{b^n : n \geq 0\}$ is $\mathbf{\Lambda} + \mathbf{b} + \mathbf{bb} + \mathbf{bbb} + \cdots$.

2. **[25 points]** For each of the following languages L over the alphabet $\Sigma = \{0, 1\}$, give a regular expression for L.

   (a) L exactly consists of all strings that end in 110.

   Regular Expression: ______________________________

   (b) L exactly consists of all strings that do not contain 00 as a substring.

   Regular Expression: ______________________________

Scratch-work area

3. **[25 points]** For each of the following languages L over the alphabet $\Sigma = \{a, b\}$, give a finite automaton that accepts exactly L.

   (a) L exactly consists of all strings that begin with b.

   Draw finite automaton here:

   (b) L exactly consists of all strings in which the number of a's is divisible by 3; i.e., the number of a's in a string $w \in L$ is $3n$ for some integer $n \geq 0$.

   Draw finite automaton here:

Scratch-work area

4. **[20 points]** For any language L, define the transpose of L, denoted by L^t, to be the language of exactly those words that are words in L spelled backward; i.e., $L^t = \{\text{reverse}(w) : w \in L\}$. For example, if $L = \{a, abb, bbaab, bbbaa\}$, then $L^t = \{a, bba, baabb, aabbb\}$. Prove that $(L_1 L_2)^t = L_2^t L_1^t$ for languages L_1 and L_2.