Midterm Exam II
CIS 341: Introduction to Logic and Automata — **Spring 2004, evening**
Prof. Marvin K. Nakayama

Print family (or last) name: ______________________________

Print given (or first) name: ______________________________

I have read and understand all of the instructions below, and I will obey the Academic Honor Code.

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Signature and Date

- This exam has 5 pages in total, numbered 1 to 5. Make sure your exam has all the pages.
- This exam will be 1 hour and 25 minutes in length.
- This is a closed-book, closed-note exam.
- For all problems, follow these instructions:
  1. Show your work and give reasons (except for question 1).
  2. Give only your answers in the spaces provided. I will only grade what you put in the answer space, and I will take off points for any scratch work in the answer space. Use the scratch-work area or the backs of the sheets to work out your answers before filling in the answer space.
  3. FA stands for finite automaton; TG stands for transition graph.
  4. For any proofs, be sure to provide a step-by-step argument, with justifications for every step.

Problem	1	2	3	4	Total
Points					

1. **[30 points]** For each of the following, circle TRUE if the statement is correct. Otherwise, circle FALSE

(a) TRUE FALSE — If L is a nonregular language, then L has a nondeterministic finite automaton.

(b) TRUE FALSE — A regular expression for the language $L = \{a^n b^n : n \geq 0\}$ is $\mathbf{a}^*\mathbf{b}^*$.

(c) TRUE FALSE — If L_1 and L_2 are languages such that $L_1 + L_2$ is regular, then L_1 and L_2 must be regular.

(d) TRUE FALSE — If language L_1 has a regular expression and language L_2 has a transition graph, then $L_1 \cap L_2$ must be regular.

(e) TRUE FALSE — If a language L has a nondeterministic finite automaton, then L' also does.

(f) TRUE FALSE — If L is infinite, then L is a nonregular language.

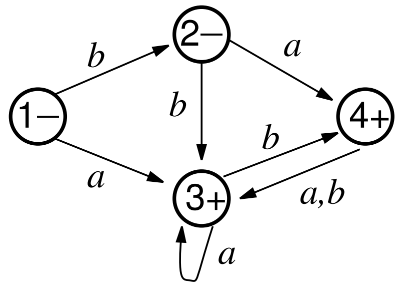
(g) TRUE FALSE — If L has a context-free grammar, then L is a nonregular language.

(h) TRUE FALSE — If a language L has a transition graph, then L^* has a regular expression.

(i) TRUE FALSE — If L_1 and L_2 are nonregular languages, then $L_1 \cap L_2$ must be nonregular.

(j) TRUE FALSE — An effective procedure to decide if a finite automaton with N states accepts any strings over an alphabet Σ is to keep testing strings over Σ on the FA until a string is accepted.

2. **[30 points]** Let L be the language of finite automata below.



Give a regular expression for L.

Answer: ____

Scratch-work area

3. **[20 points]** Suppose that a language L_1 is accepted by a finite automaton $FA_1 = (K_1, \Sigma, \pi_1, s_1, F_1)$, and a language L_2 is accepted by a finite automaton $FA_2 = (K_2, \Sigma, \pi_2, s_2, F_2)$, where for FA_i, $i = 1, 2$, K_i is the set of states, Σ is the input alphabet, $\pi_i : K_i \times \Sigma \to K_i$ is the transition function, s_i is the initial state, and F_i is the set of final states. Define the language $L_3 = L_1 \cap L_2$, and for this question, you are to define a finite automaton $FA_3 = (K_3, \Sigma, \pi_3, s_3, F_3)$ for L_3 in terms of FA_1 and FA_2, as we covered in class.

   (a) Define the set K_3 of states of FA_3 in terms of FA_1 and FA_2.

   (b) Define the transition function π_3 of FA_3 in terms of FA_1 and FA_2.

   (c) Define the initial state s_3 of FA_3 in terms of FA_1 and FA_2.

   (d) Define the set F_3 of final states of FA_3 in terms of FA_1 and FA_2.

4. **[20 points]** Recall the pumping lemma:

Theorem 14 *Let L be a language accepted by a finite automaton having N states, and let $w \in L$ with length$(w) \geq N$. Then there exists strings x, y, and z such that*

(i) $w = xyz$,

(ii) $y \neq \Lambda$,

(iii) *length*(x) + *length*$(y) \leq N$,

(iv) $xy^kz \in L$ *for all* $k = 0, 1, 2, \ldots$.

Let L be the language defined by the context-free grammar with $\Omega = \{S\}$, $\Sigma = \{a, b\}$, and productions

$$S \rightarrow aSa \mid bSb \mid a \mid b$$

Prove that L is a nonregular language.