Midterm Exam
CIS 341-451: Foundations of Computer Science II — **Fall 2005, eLearning section**
Prof. Marvin K. Nakayama

Print family (or last) name: _____

Print given (or first) name: _____

I have read and understand all of the instructions below, and I will obey the Academic Honor Code.

_____
Signature and Date

- This exam has 8 pages in total, numbered 1 to 8. Make sure your exam has all the pages.
- The exam is to be given on Saturday, October 22, 2005, 12:30-3:00pm, EST.
- This is a closed-book, closed-note exam. No calculators are allowed.
- For all problems, follow these instructions:
    1. Give only your answers in the spaces provided. I will only grade what you put in the answer space, and I will take off points for any scratch work in the answer space. Use the scratch-work area or the backs of the sheets to work out your answers before filling in the answer space.
    2. DFA stands for deterministic finite automaton; NFA stands for nondeterministic finite automaton; CFG stands for context-free grammar; PDA stands for pushdown automaton; TM stands for Turing machine.
    3. For any proofs, be sure to provide a step-by-step argument, with justifications for every step.

Problem	1	2	3	4	5	6	7	Total
Points								

1. **[20 points]** For each of the following, circle TRUE if the statement is correct. Otherwise, circle FALSE

(a) TRUE FALSE — If A is recognized by an NFA, then A is a context-free language.

(b) TRUE FALSE — Every context-free language is regular.

(c) TRUE FALSE — Every context-free language is not regular.

(d) TRUE FALSE — A language L has a CFG if and only if L is recognized by a PDA.

(e) TRUE FALSE — If a language A is not context-free, then it must be infinite.

(f) TRUE FALSE — The class of Turing-decidable languages is closed under union.

(g) TRUE FALSE — If A is a nonregular language, then A is recognized by an NFA.

(h) TRUE FALSE — A regular expression for the language $\{ 0^n1^n \mid n \geq 0 \}$ is $\varepsilon \cup 01 \cup 0011 \cup 000111 \cup \cdots$.

(i) TRUE FALSE — $\emptyset$ is a context-free language.

(j) TRUE FALSE — Every DFA is also an NFA.

2. **[20 points]** Give definitions or meanings of the following terms and phrases. **Each answer should be at most two sentences. Be sure to define any notation that you use.**

   (a) The complement of a language A defined over alphabet Σ.

   (b) Chomsky normal form.

   (c) The difference between a Turing-decidable language and a Turing-recognizable language.

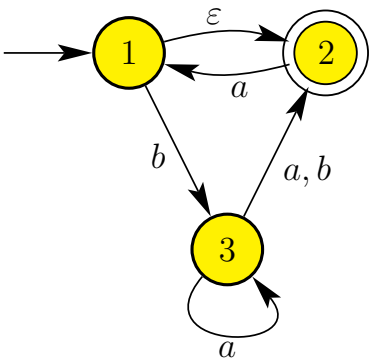
   (d) The class of context-free languages is closed under concatenation.

3. **[10 points]** Let A be the language over the alphabet $\Sigma = \{a, b\}$ defined by regular expression $(a \cup ba)^*(ab)^*$. Give an NFA that recognizes A.

Draw an NFA for A here.

Scratch-work area

4. **[10 points]** Convert the following NFA N into an equivalent DFA.



Answer:

Scratch-work area

5. **[20 points]** Consider the language $A = \{ w \in \Sigma^* \mid w = w^{\mathcal{R}} \}$, where $\Sigma = \{a, b\}$ and $w^{\mathcal{R}}$ denotes the reverse of string w.

   (a) Give a context-free grammar that describes A.

   (b) Give a pushdown automaton that recognizes A.

Scratch-work area

6. **[10 points]** Show that the collection of Turing-recognizable languages is closed under union.

7. **[10 points]** Recall the pumping lemma for context-free languages:

   Theorem: For every context-free language L, there exists a pumping length p such that, if $s \in L$ with $|s| \geq p$, then we can write $s = uvxyz$ with

   (i) $uv^i xy^i z \in L$ for each $i \geq 0$,

   (ii) $|vy| > 0$, and

   (iii) $|vxy| \leq p$.

   Prove that $A = \{\, a^{3n} b^n c^{2n} \mid n \geq 0 \,\}$ is not a context-free language.