Midterm Exam I
CIS 341: Foundations of Computer Science II - Fall 2005, day section
Prof. Marvin K. Nakayama

Print family (or last) name: $\qquad$

Print given (or first) name: $\qquad$

I have read and understand all of the instructions below, and I will obey the Academic Honor Code.

Signature and Date

- This exam has 6 pages in total, numbered 1 to 6 . Make sure your exam has all the pages.
- This exam will be 1 hour and 25 minutes in length.
- This is a closed-book, closed-note exam.
- For all problems, follow these instructions:

1. Give only your answers in the spaces provided. I will only grade what you put in the answer space, and I will take off points for any scratch work in the answer space. Use the scratch-work area or the backs of the sheets to work out your answers before filling in the answer space.
2. DFA stands for deterministic finite automaton; NFA stands for nondeterministic finite automaton.
3. For any proofs, be sure to provide a step-by-step argument, with justifications for every step.

| Problem | 1 | 2 | 3 | 4 | 5 | Total |
| :---: | :--- | :--- | :--- | :--- | :--- | :---: |
| Points |  |  |  |  |  |  |

1. [20 points] For each of the following, circle TRUE if the statement is correct. Otherwise, circle FALSE
(a) TRUE FALSE - If $A$ is a nonregular language and $B$ is a language such that $B \subseteq A$, then $B$ must be nonregular.
(b) TRUE FALSE - If $A$ is a regular language and $B$ is a language such that $B \subseteq A$, then $B$ must be regular.
(c) TRUE FALSE - A regular expression for the language $\left\{a^{n} b^{n} \mid n \geq 0\right\}$ is $\varepsilon \cup a b \cup a a b b \cup a a a b b b \cup \cdots$.
(d) TRUE FALSE - An NFA may have no accept states.
(e) TRUE FALSE - The regular expressions $\left(0^{*} 1^{*}\right)^{*}$ and $(0 \cup 1)^{*}$ generate the same language.
(f) TRUE FALSE - If $A$ is a language recognized by an NFA and $B$ is the complement of a language having a regular expression, then $\overline{(A \circ B)}$ is regular.
(g) TRUE FALSE - The class of regular languages is closed under union.
(h) TRUE FALSE - If $A$ is a regular language, then $A$ is finite.
(i) TRUE FALSE - A DFA accepts $\varepsilon$ if and only if the start state is also an accept state.
(j) TRUE FALSE - Every DFA is also an NFA.
2. [20 points] Give definitions or meanings of the following terms and phrases. Each answer should be at most two sentences. Be sure to define any notation that you use.
(a) The complement of a language $A$ defined over alphabet $\Sigma$.
(b) The transition function $\delta$ of an NFA.
(c) Nonregular language.
(d) The class of regular languages is closed under intersection.
3. [20 points] Let $A$ be the language over the alphabet $\Sigma=\{a, b\}$ defined by regular expression $\left((a b)^{*} b \cup a a\right)^{*}$. Give an NFA that recognizes $A$.

Give NFA for $A$ here.

Scratch-work area
4. [20 points] Let $\Sigma=\{0,1\}$. A string over $\Sigma$ is said to contain a double symbol if it contains either 00 or 11 as a substring. Give a regular expression for each of the languages below.
(a) $A=\left\{w \in \Sigma^{*} \mid w\right.$ does not end in a double symbol $\}$.

## Answer:

(b) $B=\left\{w \in \Sigma^{*} \mid w\right.$ contains exactly one double symbol $\}$. (The string 10010 has exactly one double symbol, but 100010 has two double symbols.)

Answer:

## Scratch-work area

5. [20 points] Recall the pumping lemma:

Theorem: If $A$ is a regular language, then $\exists$ number $p$ (pumping length) where, if $s \in A$ with $|s| \geq p$, then $\exists$ strings $x, y, z$ such that $s=x y z$ and
(i) $x y^{i} z \in A$ for each $i \geq 0$,
(ii) $|y|>0$, and
(iii) $|x y| \leq p$.

Prove that $C=\left\{w w \mid w \in \Sigma^{*}\right\}$ is a nonregular language, where $\Sigma=\{0,1\}$.

