Midterm Exam I CIS 341: Foundations of Computer Science II — Fall 2005, day section Prof. Marvin K. Nakayama

Print family (or last) name: \_\_\_\_\_

Print given (or first) name:

I have read and understand all of the instructions below, and I will obey the Academic Honor Code.

Signature and Date

- This exam has 6 pages in total, numbered 1 to 6. Make sure your exam has all the pages.
- This exam will be 1 hour and 25 minutes in length.
- This is a closed-book, closed-note exam.
- For all problems, follow these instructions:
  - 1. Give only your answers in the spaces provided. I will only grade what you put in the answer space, and I will take off points for any scratch work in the answer space. Use the scratch-work area or the backs of the sheets to work out your answers before filling in the answer space.
  - 2. DFA stands for deterministic finite automaton; NFA stands for nondeterministic finite automaton.
  - 3. For any proofs, be sure to provide a step-by-step argument, with justifications for every step.

Problem	1	2	3	4	5	Total
Points						

- 1. **[20 points]** For each of the following, circle TRUE if the statement is correct. Otherwise, circle FALSE
  - (a) TRUE FALSE If A is a nonregular language and B is a language such that  $B \subseteq A$ , then B must be nonregular.
  - (b) TRUE FALSE If A is a regular language and B is a language such that  $B \subseteq A$ , then B must be regular.
  - (c) TRUE FALSE A regular expression for the language  $\{a^n b^n \mid n \ge 0\}$ is  $\varepsilon \cup ab \cup aabb \cup aaabb \cup \cdots$ .
  - (d) TRUE FALSE An NFA may have no accept states.
  - (e) TRUE FALSE The regular expressions  $(0^*1^*)^*$  and  $(0 \cup 1)^*$  generate the same language.
  - (f) TRUE FALSE If A is a language recognized by an NFA and B is the complement of a language having a regular expression, then  $\overline{(A \circ B)}$  is regular.
  - (g) TRUE FALSE The class of regular languages is closed under union.
  - (h) TRUE FALSE If A is a regular language, then A is finite.
  - (i) TRUE FALSE A DFA accepts  $\varepsilon$  if and only if the start state is also an accept state.
  - (j) TRUE FALSE Every DFA is also an NFA.

- 2. [20 points] Give definitions or meanings of the following terms and phrases. Each answer should be at most two sentences. Be sure to define any notation that you use.
  - (a) The complement of a language A defined over alphabet  $\Sigma$ .

(b) The transition function  $\delta$  of an NFA.

(c) Nonregular language.

(d) The class of regular languages is closed under intersection.

3. [20 points] Let A be the language over the alphabet  $\Sigma = \{a, b\}$  defined by regular expression  $((ab)^*b \cup aa)^*$ . Give an NFA that recognizes A.

Give NFA for A here.

Scratch-work area

- 4. [20 points] Let  $\Sigma = \{0, 1\}$ . A string over  $\Sigma$  is said to contain a double symbol if it contains either 00 or 11 as a substring. Give a regular expression for each of the languages below.
  - (a)  $A = \{ w \in \Sigma^* \mid w \text{ does not end in a double symbol } \}.$

## Answer:

(b)  $B = \{ w \in \Sigma^* \mid w \text{ contains exactly one double symbol} \}$ . (The string 10010 has exactly one double symbol, but 100010 has two double symbols.)

Answer:

## Scratch-work area

## 5. [20 points] Recall the pumping lemma:

**Theorem:** If A is a regular language, then  $\exists$  number p (pumping length) where, if  $s \in A$  with  $|s| \ge p$ , then  $\exists$  strings x, y, z such that s = xyz and

- (i)  $xy^i z \in A$  for each  $i \ge 0$ ,
- (ii) |y| > 0, and
- (iii)  $|xy| \leq p$ .

Prove that  $C = \{ ww \mid w \in \Sigma^* \}$  is a nonregular language, where  $\Sigma = \{0, 1\}$ .