Midterm Exam II
CIS 341: Foundations of Computer Science II — **Fall 2005, day section**
Prof. Marvin K. Nakayama

Print family (or last) name: ______________________________

Print given (or first) name: ______________________________

I have read and understand all of the instructions below, and I will obey the Academic Honor Code.

Signature and Date: ______________________________

- This exam has 6 pages in total, numbered 1 to 6. Make sure your exam has all the pages.
- This exam will be 1 hour and 25 minutes in length.
- This is a closed-book, closed-note exam.
- For all problems, follow these instructions:
  1. Give only your answers in the spaces provided. I will only grade what you put in the answer space, and I will take off points for any scratch work in the answer space. Use the scratch-work area or the backs of the sheets to work out your answers before filling in the answer space.
  2. DFA stands for deterministic finite automaton; NFA stands for nondeterministic finite automaton; CFG stands for context-free grammar; PDA stands for pushdown automaton.
  3. For any proofs, be sure to provide a step-by-step argument, with justifications for every step. Unless you are specifically asked to prove a theorem from the book, you may assume that the theorems in the textbook hold; i.e., you do not have to reprove the theorems in the textbook. When using a theorem from the textbook, make sure you provide enough detail so that it is clear which result you are using; e.g., say something like, "By the theorem that shows every NFA has an equivalent DFA, it follows that . . . "

Problem	1	2	3	4	5	Total
Points						

1. **[20 points]** For each of the following, circle TRUE if the statement is correct. Otherwise, circle FALSE

(a) TRUE FALSE — If language A has a CFG, then A has a regular expression.

(b) TRUE FALSE — If language A has a regular expression, then A has a CFG.

(c) TRUE FALSE — A Turing machine $M = (Q, \Sigma, \Gamma, \delta, q_0, q_{\text{accept}}, q_{\text{reject}})$ will either accept or reject any string $w \in \Sigma^*$.

(d) TRUE FALSE — There is a language A that is recognized by a 3-tape Turing machine but is not recognized by any 1-tape Turing machine.

(e) TRUE FALSE — Every Turing-recognizable language is also Turing-decidable.

(f) TRUE FALSE — If language A has a PDA, then A has a CFG in Chomsky normal form.

(g) TRUE FALSE — The language $A = \{\, ww \mid w \in \Sigma^* \}$ with $\Sigma = \{0, 1\}$ is context-free.

(h) TRUE FALSE — If a language is Turing-decidable, then it is also context-free.

(i) TRUE FALSE — There is an algorithm to determine if a regular expression R generates a string w.

(j) TRUE FALSE — Every nondeterministic Turing machine has an equivalent deterministic Turing machine.

2. **[24 points]** Give a short answer (at most two sentences) for each part below. No proofs are required, but be sure to define any notation that you use.

   (a) What is a configuration of a Turing machine?

   (b) Give the language corresponding to the DFA acceptance problem.

   (c) Give a CFG for the language $L = \{ b^n a^n \mid n \geq 1 \}$.

   (d) What is the Church-Turing Thesis?

3. [**10 points**] Show that the collection of Turing-decidable languages is closed under union.

4. **[26 points]** Consider the following CFG $G = (V, \Sigma, R, S)$, where $V = \{S, T, X\}$, $\Sigma = \{a, b\}$, the start variable is S, and the rules R are

$$\begin{aligned} S &\rightarrow TX \\ T &\rightarrow TSS \mid a \\ X &\rightarrow b \mid \varepsilon \end{aligned}$$

Give a PDA that recognizes the language $L(G)$.

Give PDA here.

Scratch-work area

5. **[20 points]** Let $A\varepsilon_{\text{CFG}} = \{\, \langle G \rangle \mid G \text{ is a CFG that generates } \varepsilon \,\}$. Show that $A\varepsilon_{\text{CFG}}$ is decidable.