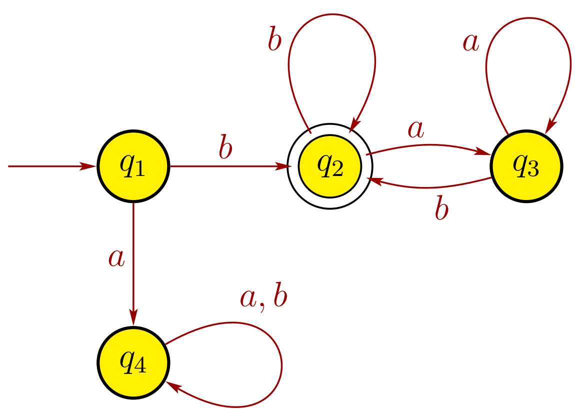
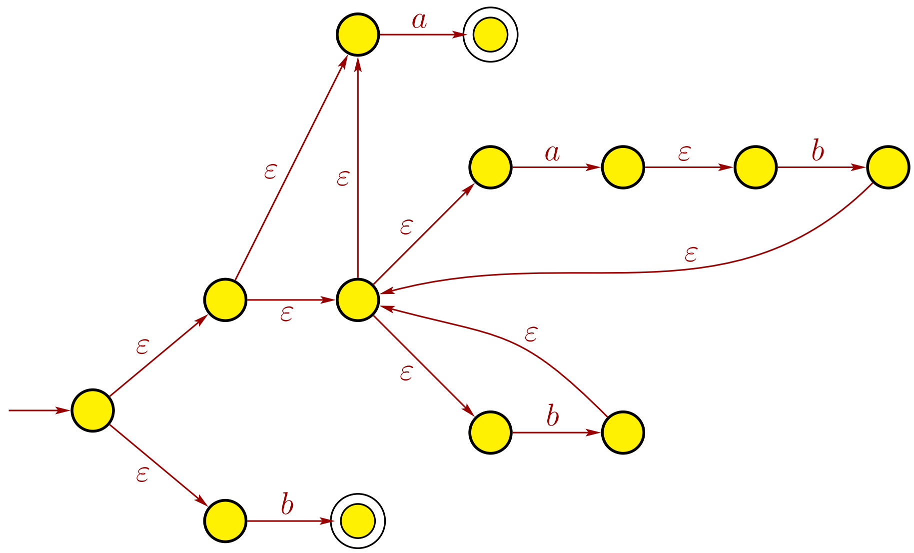
CS 341, Fall 2006
Solutions for Midterm, eLearning Section

1. (a) False. $A = \{0^n1^n \mid n \geq 0\}$ is a context-free language, but it is nonregular. Hence, A cannot have an NFA.
   (b) True. Homework 5, problem 3(b).
   (c) True. Homework 5, problem 3(a).
   (d) True. Theorem 2.20.
   (e) True. If A is finite, then it must be regular (page 1-81 of notes).
   (f) False. $\{0,1\}^*$ is a regular language that is infinite.
   (g) True, by Kleene's Theorem.
   (h) False. 0^*1^* generates $00111 \notin \{0^n1^n \mid n \geq 0\}$.
   (i) False. $A \times B$ contains pairs of strings, whereas $A \circ B$ contains just strings.
   (j) True. page 1-50 of notes.

2. (a) Lexicographic order means that shorter strings appear before longer strings, and strings of the same length are in alphabetical order.
   (b) The difference is in the transition function δ. For a DFA, $\delta : Q \times \Sigma \to Q$, where Q is the set of states and Σ is the alphabet. For an NFA, $\delta : Q \times \Sigma_\varepsilon \to P(Q)$, where $\Sigma_\varepsilon = \Sigma \cup \{\varepsilon\}$ and $P(Q)$ is the power set of Q.
   (c) $b \cup b(a \cup b)^*b$
   (d) DFA



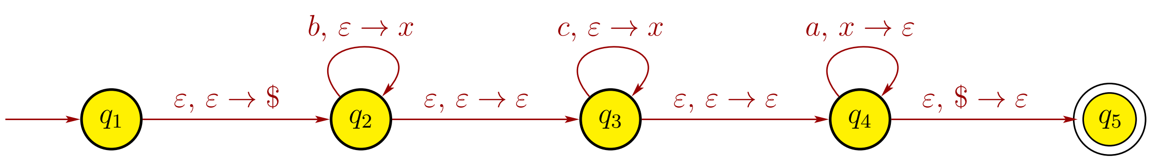
3. NFA



4. $(a \cup ba^*b)(ba \cup (bb \cup a)a^*b)$

5. (a) $G = (V, \Sigma, R, S)$, where $V = \{S, X\}$, $\Sigma = \{a, b, c\}$, and the rules are $S \to bSa \mid X$, $X \to cXa \mid \varepsilon$.

   (b) PDA



6. No, the class of context-free languages is not closed under intersection. Consider the languages $A = \{a^n b^n c^k \mid n, k \geq 0\}$ and $B = \{a^n b^k c^n \mid n, k \geq 0\}$. A CFG for A is $G_1 = (V_1, \Sigma, R_1, S_1)$, with $V_1 = \{S_1, X, Y\}$, $\Sigma = \{a, b, c\}$, and rules $S_1 \to XY$, $X \to aXb \mid \varepsilon$, $Y \to cY \mid \varepsilon$. A CFG for B is $G_2 = (V_2, \Sigma, R_2, S_2)$, with $V_2 = \{S_2, Z\}$, $\Sigma = \{a, b, c\}$, and rules $S_2 \to aS_2c \mid Z$, $Z \to bZ \mid \varepsilon$. Then $A \cap B = \{a^n b^n c^n \mid n \geq 0\}$, which is not context-free (see page 2-105 of the notes).

7. Suppose that $A = \{a^{3n} b^n c^{2n} \mid n \geq 0\}$ is a regular language. Let p be the pumping length, and consider the string $s = a^{3p} b^p c^{2p} \in A$. Note that $|s| = 6p \geq p$, so the pumping lemma implies we can write $s = xyz$ with $xy^i z \in A$ for all $i \geq 0$, $|y| > 0$, and $|xy| \leq p$. Now, $|xy| \leq p$ implies that x and y have only a's (together up to p

in total) and z has the rest of the a's followed by $b^p c^{2p}$. Hence, we can write $x = a^j$ for some $j \geq 0$, $y = a^k$ for some $k \geq 0$, and $z = a^\ell b^p c^{2p}$, where $j + k + \ell = 3p$ since $xyz = s = a^{3p} b^p c^{2p}$. Also, $|y| > 0$ implies $k > 0$. Now consider the string $xyyz = a^j a^k a^k a^\ell b^p c^{2p} = a^{3p+k} b^p c^{2p}$ since $j + k + \ell = 3p$. Note that $xyyz \notin A$, which is a contradiction, so A is not a regular language.