Midterm Exam
CS 341-451: Foundations of Computer Science II — **Fall 2006, eLearning section**
Prof. Marvin K. Nakayama

Print family (or last) name: _____

Print given (or first) name: _____

I have read and understand all of the instructions below, and I will obey the Academic Honor Code.

_____
Signature and Date

- This exam has 8 pages in total, numbered 1 to 8. Make sure your exam has all the pages.
- The exam is to be given on Sunday, October 22, 2006, 12:30-3:00pm, EST.
- This is a closed-book, closed-note exam. No calculators are allowed.
- For all problems, follow these instructions:
    1. Give only your answers in the spaces provided. I will only grade what you put in the answer space, and I will take off points for any scratch work in the answer space. Use the scratch-work area or the backs of the sheets to work out your answers before filling in the answer space.
    2. DFA stands for deterministic finite automaton; NFA stands for nondeterministic finite automaton; CFG stands for context-free grammar; PDA stands for pushdown automaton.
    3. For any proofs, be sure to provide a step-by-step argument, with justifications for every step.

Problem	1	2	3	4	5	6	7	Total
Points								

1. **[20 points]** For each of the following, circle TRUE if the statement is correct. Otherwise, circle FALSE

(a) TRUE FALSE — If A is a context-free language, then A is recognized by an NFA.

(b) TRUE FALSE — If A and B are context-free languages, then so is $A \circ B$.

(c) TRUE FALSE — If A and B are context-free languages, then so is $A \cup B$.

(d) TRUE FALSE — A language L has a CFG if and only if L is recognized by a PDA.

(e) TRUE FALSE — If a language A is not regular, then it must be infinite.

(f) TRUE FALSE — If a language is infinite, then it must not be regular.

(g) TRUE FALSE — If A is a regular language, then A has a regular expression.

(h) TRUE FALSE — A regular expression for the language $\{ 0^n 1^n \mid n \geq 0 \}$ is 0^*1^*.

(i) TRUE FALSE — If $A = \{01, 1\}$ and $B = \{\varepsilon\}$, then $A \times B = A \circ B$.

(j) TRUE FALSE — The class of regular languages is closed under concatenation.

2. **[20 points]** Give short answers to each of the following parts. **Each answer should be at most three sentences. Be sure to define any notation that you use.**

   (a) What does it mean for a set A of strings to be in lexicographic order?

   (b) Explain the difference between a DFA and an NFA.

   (c) Give a regular expression for the language consisting of strings over the alphabet $\Sigma = \{a, b\}$ that start and end with b.

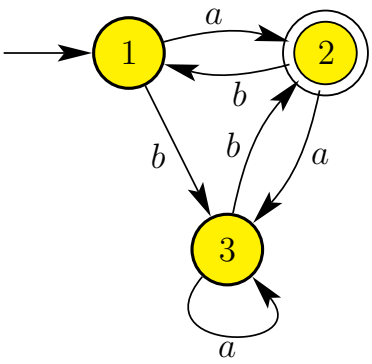
   (d) Give a DFA for the language consisting of strings over the alphabet $\Sigma = \{a, b\}$ that start and end with b. You only need to draw the graph; do not specify the DFA as a 5-tuple.

3. **[10 points]** Let A be the language over the alphabet $\Sigma = \{a, b\}$ defined by regular expression $(ab \cup b)^* a \cup b$. Give an NFA that recognizes A.

Draw an NFA for A here.

Scratch-work area

4. **[10 points]** For the DFA M below, give a regular expression for $L(M)$.



Answer:

Scratch-work area

5. **[20 points]** Consider the language $A = \{ b^i c^j a^k \mid i, j, k \geq 0 \text{ and } i + j = k \}$.

   (a) Give a context-free grammar G that describes A. Be sure to specify G as a 4-tuple $G = (V, \Sigma, R, S)$.

   (b) Give a pushdown automaton that recognizes A. You only need to draw the picture.

Scratch-work area

6. **[10 points]** Is the class of context-free languages closed under intersection?

   Circle one: YES NO

   - If YES, give a proof.
   - If NO, give an example of two context-free languages A and B whose intersection is not context-free. Also, give the rules of the context-free grammars for A and B. You do not need to prove that the intersection of A and B is not context-free.

7. **[10 points]** Recall the pumping lemma for regular languages:

   Theorem: For every regular language L, there exists a pumping length p such that, if $s \in L$ with $|s| \geq p$, then we can write $s = xyz$ with

   (i) $xy^iz \in L$ for each $i \geq 0$,

   (ii) $|y| > 0$, and

   (iii) $|xy| \leq p$.

   Prove that $A = \{\, a^{3n}b^nc^{2n} \mid n \geq 0 \,\}$ is not a regular language.