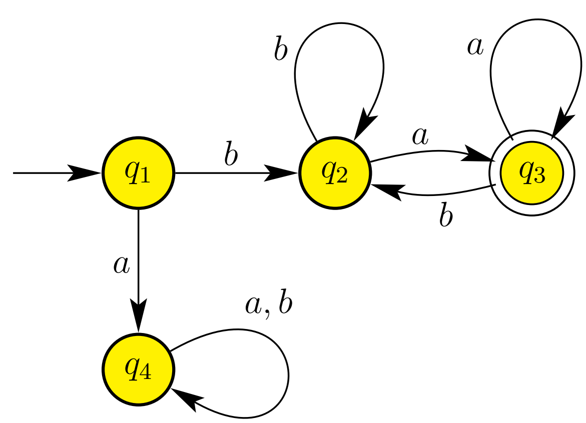
CS 341, Fall 2006
Solutions for Midterm I

1. (a) False. The empty language $\emptyset$ has no strings at all, not even the empty string ε.
   (b) False. $\emptyset$ is a language and ε is a string, so they can't be equal.
   (c) False. $L(R \circ \emptyset) = \emptyset \neq L(R)$ if $R \neq \emptyset$.
   (d) True, by Corollary 1.40.
   (e) True, by Corollary 1.40.
   (f) False. The language $\{a, b\}^*$ is infinite and has a DFA (see lecture notes 1-18), so it is regular.
   (g) False. The string 0 begins and ends with 0 but cannot be generated by R.
   (h) True, by Homework 2, problem 5.
   (i) False. $\emptyset^* = \{\varepsilon\} \neq \emptyset$.
   (j) False. $R \circ \varepsilon = R$ by lecture notes 1-62, so $L(R \circ \varepsilon) \neq \emptyset$ when $L(R) \neq \emptyset$.

2. (a) $A \times B = \{(11, \varepsilon), (11, 1), (111, \varepsilon), (111, 1)\}$ and $A \circ B = \{11, 111, 1111\}$.
   (b) There are many sets S for which $S^* = S^+$, e.g., $S = \{\varepsilon\}$. In fact, $S^* = S^+$ if and only if $\varepsilon \in S$.
   (c) There are many sets S for which $S^* = S$, e.g., $S = \{\varepsilon\}$.
   (d) The difference between a DFA and an NFA is in the transition function δ. For a DFA, $\delta : Q \times \Sigma \to Q$. For an NFA, $\delta : Q \times \Sigma_\varepsilon \to P(Q)$, where $P(Q)$ is the power set of Q.

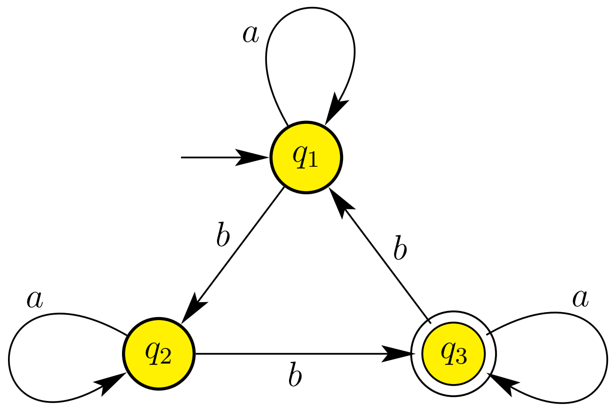
3. (a) Strings that begin with b and end with a.
   DFA



   Regular expression: $b(a \cup b)^*a$

(b) Strings w such that $n_b(w) \bmod 3 = 2$

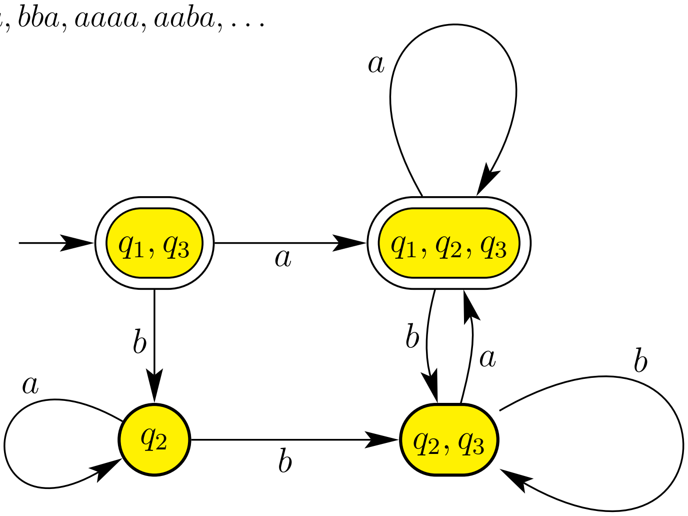
DFA



Regular expression: $a^*ba^*b(ba^*ba^*b \cup a)^*$

4. (a) $\varepsilon, a, aa, aaa, aba, bba, aaaa, aaba, \ldots$

(b)



5. We prove this by contradiction. Suppose that $\overline{M}$ is not a minimal DFA for $\overline{A}$. Then there exists another DFA D for $\overline{A}$ such that D has strictly fewer states than $\overline{M}$. Now create another DFA D' by swapping the accepting and non-accepting states of D. Then D' recognizes the complement of $\overline{A}$. But the complement of $\overline{A}$ is just A, so D' recognizes A. Note that D' has the same number of states as D, and $\overline{M}$ has the same number of states as M. Thus, since we assumed that D has strictly fewer states than $\overline{M}$, then D' has strictly fewer states than M. But since D' recognizes A, this contradicts our assumption that M is a minimal DFA for A. Therefore, $\overline{M}$ is a minimal DFA for $\overline{A}$.