Midterm Exam I
CS 341: Foundations of Computer Science II — **Fall 2006, day section**
Prof. Marvin K. Nakayama

Print family (or last) name: _____

Print given (or first) name: _____

I have read and understand all of the instructions below, and I will obey the Academic Honor Code.

_____
Signature and Date

- This exam has 6 pages in total, numbered 1 to 6. Make sure your exam has all the pages.
- Note the number written on the upper right-hand corner of the first page. On the sign-up sheet being passed around, sign your name next to this number.
- This exam will be 1 hour and 25 minutes in length.
- This is a closed-book, closed-note exam.
- For all problems, follow these instructions:
    1. Give only your answers in the spaces provided. I will only grade what you put in the answer space, and I will take off points for any scratch work in the answer space. Use the scratch-work area or the backs of the sheets to work out your answers before filling in the answer space.
    2. DFA stands for deterministic finite automaton; NFA stands for nondeterministic finite automaton.
    3. For any proofs, be sure to provide a step-by-step argument, with justifications for every step.

Problem	1	2	3	4	5	Total
Points						

1. **[20 points]** For each of the following, circle TRUE if the statement is correct. Otherwise, circle FALSE

(a) TRUE FALSE — $\emptyset = \{\varepsilon\}$.

(b) TRUE FALSE — $\emptyset = \varepsilon$.

(c) TRUE FALSE — If R is any regular expression, then $L(R \circ \emptyset) = L(R)$.

(d) TRUE FALSE — If A is recognized by an NFA, then A is regular.

(e) TRUE FALSE — If A is a regular language, then there is an NFA that recognizes A.

(f) TRUE FALSE — If A is a regular language, then $|A| < \infty$.

(g) TRUE FALSE — If regular expression $R = 0(0 \cup 1)^*0$, then $L(R)$ is the language of all strings over $\Sigma = \{0, 1\}$ that begin and end with 0.

(h) TRUE FALSE — The class of regular languages is closed under intersection.

(i) TRUE FALSE — $\emptyset^* = \emptyset$.

(j) TRUE FALSE — If R is any regular expression, then $L(R \circ \varepsilon) = \emptyset$.

2. **[20 points]** Give short answers to each of the following parts. Each answer should be at most three sentences. Be sure to define any notation that you use.

   (a) For the sets $A = \{11, 111\}$ and $B = \{\epsilon, 1\}$, what are $A \times B$ and $A \circ B$?

   (b) Give an example of a set S such that $S^* = S^+$.

   (c) Give an example of a set S such that $S^* = S$.

   (d) Explain the difference between a DFA and an NFA.

3. **[20 points]** For each of the following languages over the alphabet $\Sigma = \{a, b\}$, give a DFA and a regular expression for it. For the DFA, you only need to draw the graph; you do not need to formally define it as a 5-tuple. Also, for any string $w \in \Sigma^*$, define $n_b(w)$ to be the number of b's in w.

   (a) All strings that begin with b and end with a.
   Draw DFA here:

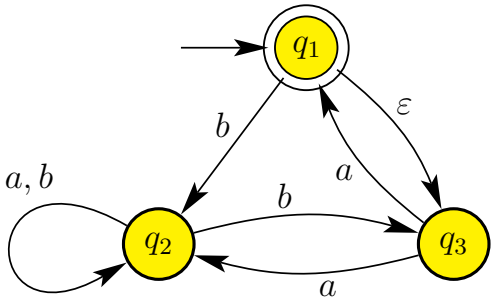
   Give regular expression here:

   (b) All strings w such that $n_b(w) \bmod 3 = 2$.
   Draw DFA here:

   Give regular expression here:

Scratch-work area

4. **[25 points]** Let N be the following NFA with $\Sigma = \{a, b\}$, and let $C = L(N)$.



(a) List the strings in C in lexicographic order. If C has more than 8 strings, list only the first 8 strings in C, followed by 3 dots.

(b) Give a DFA for C.

Scratch-work area

5. **[15 points]** We say that a DFA M for a language A is *minimal* if there does not exist another DFA M' for A such that M' has strictly fewer states than M. Suppose that $M = (Q, \Sigma, \delta, q_0, F)$ is a minimal DFA for A. Using M, we construct a DFA $\overline{M}$ for the complement $\overline{A}$ as $\overline{M} = (Q, \Sigma, \delta, q_0, Q - F)$. Prove that $\overline{M}$ is a minimal DFA for $\overline{A}$.