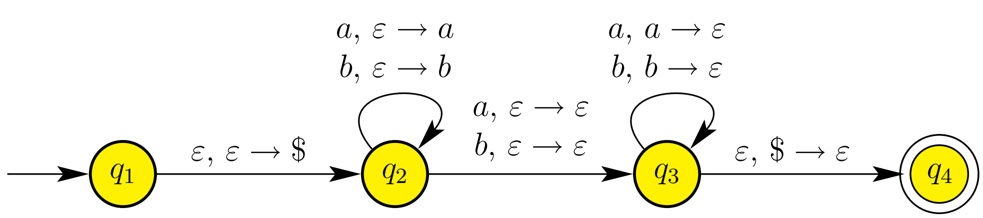
CS 341, Fall 2006
Solutions for Midterm II, Day Section

1. (a) False. Corollary 1.40.
   (b) True. Corollary 2.32.
   (c) True. Homework 5, problem 3(b).
   (d) False. All regular languages are also context-free by Corollary 2.32.
   (e) False. It is nonregular (see notes 1-90), so it cannot have a regular expression by Kleene's Theorem.
   (f) False. The language $\{a, b\}^*$ is infinite and has a DFA (see lecture notes 1-18), so it is regular.
   (g) False. Homework 6, problem 2(a).
   (h) False. A is finite, so it is regular by page 1-81 of the notes.
   (i) True. B^* is context-free by Homework 5, problem 3(c). A is context-free by Corollary 2.32, so $A \cup B^*$ is context-free by Homework 5, problem 3(a).
   (j) False. $\{a^n b^n c^n \mid n \geq 0\}$ is neither regular nor context-free.

2. (a) A CFG is in Chomsky normal form if each of its rules has one of the following 3 forms: $A \to BC$ or $A \to x$ or $S \to \varepsilon$, where A, B, C, S are variables; B and C are not the start variable S; and x is a terminal.
   (b) $\delta : Q \times \Sigma_\varepsilon \times \Gamma_\varepsilon \to P(Q \times \Gamma_\varepsilon)$, where Q is the set of states, Σ is the input alphabet, Γ is the stack alphabet, $\Sigma_\varepsilon = \Sigma \cup \{\varepsilon\}$, $\Gamma_\varepsilon = \Gamma \cup \{\varepsilon\}$, and $P(Q \times \Gamma_\varepsilon)$ denotes the power set of $Q \times \Gamma_\varepsilon$.
   (c) $G_3 = (V_3, \Sigma, R_3, S_3)$, where $V_3 = V_1 \cup V_2 \cup \{S_3\}$ and $R_3 = R_1 \cup R_2 \cup \{S_3 \to S_1 \mid S_2\}$.
   (d) See page 1-66 of notes.
   (e) See page 1-67 of notes.

3. (a) $G = (V, \Sigma, R, S)$, where $V = \{S\}$, $\Sigma = \{a, b\}$, and the rules are $S \to aSa \mid bSb \mid a \mid b$.
   (b) PDA



4. $(a \cup bb^*a)(ba \cup (a \cup bb)b^*a)^*$

5. Suppose A is context-free. Let $p \geq 1$ be the pumping length, and consider the string $s = c^p a^p b^p \in A$. Note that $|s| = 3p \geq p$, so the conclusions of the pumping lemma must hold; i.e., we can write $s = uvxyz$ with $uv^ixy^iz \in A$ for all $i \geq 0$, $|vy| \geq 1$, and $|vxy| \leq p$. Since $|vxy| \leq p$, vxy can span at most 2 types of symbols. Let's consider all of the possibilities for vxy:

   - v and y are both uniform (i.e., v contains at most one type of symbol, and y contains at most one type of symbol). Since $|vy| \geq 1$, v and y cannot both be empty. Thus, for the string uv^2xy^2z, we have increased the number of symbols of at least one type and at most two types, so since there are 3 types of symbols, there must be more of one type of symbol than another type. Hence, uv^2xy^2z does not have the same number of c's, a's and b's, so $uv^2xy^2z \notin A$.
   - v and y are not both uniform (i.e., v contains two types of symbols, or y contains two types of symbols). Then uv^2xy^2z does not have all of the c's before all of the a's and all of the a's before all of the b's. Hence, $uv^2xy^2z \notin A$.

   Thus, all possibilities lead to a contradiction, so A must not be context-free.