Midterm Exam II
CIS 341: Foundations of Computer Science II — **Fall 2006, day section**
Prof. Marvin K. Nakayama

Print family (or last) name: ______________________________

Print given (or first) name: ______________________________

I have read and understand all of the instructions below, and I will obey the Academic Honor Code.

Signature and Date: ______________________________

- This exam has 7 pages in total, numbered 1 to 7. Make sure your exam has all the pages.
- This exam will be 1 hour and 25 minutes in length.
- This is a closed-book, closed-note exam.
- For all problems, follow these instructions:
  1. Give only your answers in the spaces provided. I will only grade what you put in the answer space, and I will take off points for any scratch work in the answer space. Use the scratch-work area or the backs of the sheets to work out your answers before filling in the answer space.
  2. DFA stands for deterministic finite automaton; NFA stands for nondeterministic finite automaton; CFG stands for context-free grammar; PDA stands for pushdown automaton.
  3. For any proofs, be sure to provide a step-by-step argument, with justifications for every step. Unless you are specifically asked to prove a theorem from the book, you may assume that the theorems in the textbook hold; i.e., you do not have to reprove the theorems in the textbook. When using a theorem from the textbook, make sure you provide enough detail so that it is clear which result you are using; e.g., say something like, "By the theorem that states $S^{**} = S^*$, it follows that ..."

Problem	1	2	3	4	5	Total
Points						

1. **[20 points]** For each of the following, circle TRUE if the statement is correct. Otherwise, circle FALSE

(a) TRUE FALSE — If $A = L(N)$ for some NFA N, then A is nonregular.

(b) TRUE FALSE — If a language A has a DFA, then A is context-free.

(c) TRUE FALSE — The class of context-free languages is closed under concatenation.

(d) TRUE FALSE — If a language B is regular, then B cannot be context-free.

(e) TRUE FALSE — The language $\{ 0^n 1^n \mid n \geq 0 \}$ has a regular expression.

(f) TRUE FALSE — If a language L is regular, then L is finite.

(g) TRUE FALSE — If A and B are context-free languages, then $A \cap B$ is a context-free language.

(h) TRUE FALSE — For the alphabet $\Sigma = \{0, 1\}$, the language

$$A = \{ w \in \Sigma^* \mid w = w^{\mathcal{R}},\ |w| \leq 100 \}$$

is not regular.

(i) TRUE FALSE — If a language A is regular and a language B is context-free, then $A \cup B^*$ is context-free.

(j) TRUE FALSE — Every language is regular or context-free.

2. **[30 points]** Give a short answer (at most three sentences) for each part below. Be sure to define any notation that you use.

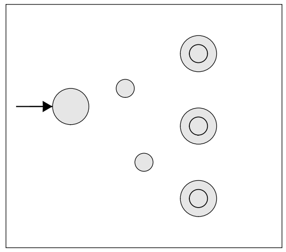
   (a) What does it mean for a context-free grammar to be in Chomsky normal form?

   (b) Give the formal definition of the transition function δ of a PDA.

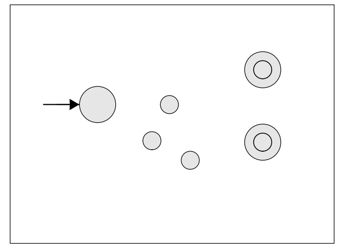
   (c) Suppose that language A_1 has CFG $G_1 = (V_1, \Sigma, R_1, S_1)$, and language A_2 has CFG $G_2 = (V_2, \Sigma, R_2, S_2)$. Give a CFG G_3 for $A_1 \cup A_2$ in terms of G_1 and G_2. You do not have to prove the correctness of your CFG G_3.

(d) Suppose that language A_1 is recognized by NFA N_1 below, and language A_2 is recognized by NFA N_2 below. Note that the transitions are not drawn in N_1 and N_2. Draw a picture of an NFA for $A_1 \circ A_2$.

N_1

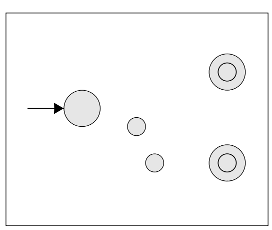


N_2



(e) Suppose that language A is recognized by the NFA N below. Note that the transitions are not drawn in N. Draw a picture of an NFA for A^*.

N



3. **[20 points]** Consider the alphabet $\Sigma = \{a, b\}$ and the language

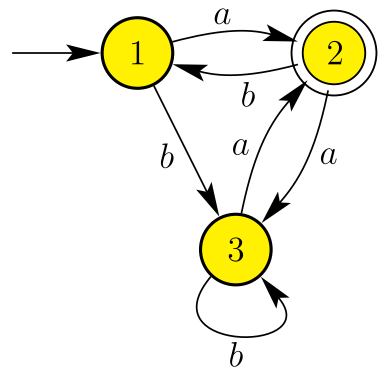
$$L = \{ \, w \in \Sigma^* \mid w = w^{\mathcal{R}} \text{ and } |w| \text{ is odd } \}.$$

(a) Give a context-free grammar G for L. Be sure to specify G as a 4-tuple $G = (V, \Sigma, R, S)$.

(b) Give a PDA for L. You only need to draw the graph.

Scratch-work area

4. **[15 points]** For the DFA M below, give a regular expression for $L(M)$.



Answer:

Scratch-work area

5. **[15 points]** Recall the pumping lemma for context-free languages:

Theorem: For every context-free language L, there exists a pumping length p such that, if $s \in L$ with $|s| \geq p$, then we can write $s = uvxyz$ with

(i) $uv^ixy^iz \in L$ for each $i \geq 0$,

(ii) $|vy| \geq 1$, and

(iii) $|vxy| \leq p$.

Prove that $A = \{ c^n a^n b^n \mid n \geq 0 \}$ is not a context-free language.