

Midterm Exam I  
CIS 341: Foundations of Computer Science II — **Spring 2006, day section**  
Prof. Marvin K. Nakayama

Print family (or last) name: \_\_\_\_\_

Print given (or first) name: \_\_\_\_\_

I have read and understand all of the instructions below, and I will obey the Academic Honor Code.

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Signature and Date

- This exam has 6 pages in total, numbered 1 to 6. Make sure your exam has all the pages.
- This exam will be 1 hour and 25 minutes in length.
- This is a closed-book, closed-note exam.
- For all problems, follow these instructions:
  1. Give only your answers in the spaces provided. I will only grade what you put in the answer space, and I will take off points for any scratch work in the answer space. Use the scratch-work area or the backs of the sheets to work out your answers before filling in the answer space.
  2. DFA stands for deterministic finite automaton; NFA stands for nondeterministic finite automaton.
  3. For any proofs, be sure to provide a step-by-step argument, with justifications for every step.

Problem	1	2	3	4	5	Total
Points						

1. [20 points] For each of the following, circle TRUE if the statement is correct. Otherwise, circle FALSE

- (a) TRUE FALSE — If  $A$  is a regular language, then  $A$  is finite.
- (b) TRUE FALSE — If  $R$  is any regular expression, then  $L(R \cup \varepsilon) = L(R)$ .
- (c) TRUE FALSE — If  $R$  is any regular expression, then  $L(R \circ \emptyset) = L(R)$ .
- (d) TRUE FALSE — A regular expression for the language  $\{b^n a^n \mid n \geq 0\}$  is  $\varepsilon \cup ba \cup bbaa \cup bbbaaa \cup \dots$ .
- (e) TRUE FALSE — If language  $A$  has a regular expression, then  $\overline{A}$  is recognized by some NFA.
- (f) TRUE FALSE — If  $A$  is a nonregular language and  $B$  is a finite language, then  $A \cup B$  must be nonregular.
- (g) TRUE FALSE — If  $A$  is a nonregular language, then there exists a regular language  $B$  with  $B \subseteq A$ .
- (h) TRUE FALSE — A DFA  $M = (Q, \Sigma, \delta, q_0, F)$  may have  $F = Q$ .
- (i) TRUE FALSE — A DFA  $M = (Q, \Sigma, \delta, q_0, F)$  may have  $F = \emptyset$ .
- (j) TRUE FALSE — If  $A$  is a regular language, then  $A^*$  must be nonregular.

2. [20 points] Give definitions or meanings of the following terms and phrases. Each answer should be at most two sentences. Be sure to define any notation that you use.

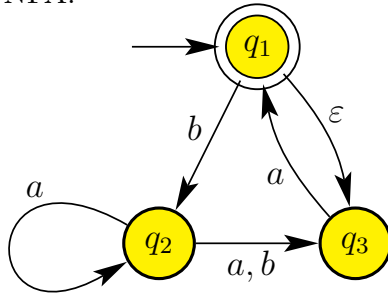
(a) For a machine  $M$ , what is  $L(M)$ ?

(b) What does Kleene's Theorem say?

(c) Define the following statement: "The class of regular languages is closed under concatenation."

(d) Explain the difference between a DFA and an NFA.

3. [20 points] Let  $A$  be the language over the alphabet  $\Sigma = \{a, b\}$  defined by the following NFA:

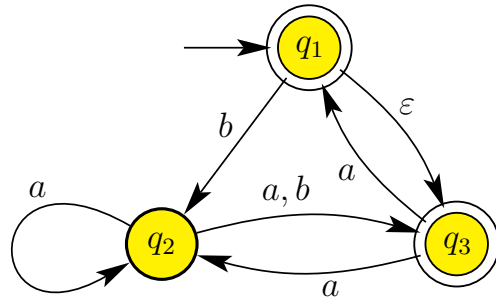


Draw here a DFA that recognizes  $A$ .

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Scratch-work area

4. [20 points] Let  $B$  be the language recognized by the following NFA:



Give a regular expression for  $B$ .

**Answer:**



Scratch-work area

5. [20 points] Recall the pumping lemma:

**Theorem:** If  $A$  is a regular language, then  $\exists$  number  $p$  (pumping length) where, if  $s \in A$  with  $|s| \geq p$ , then  $\exists$  strings  $x, y, z$  such that  $s = xyz$  and

(i)  $xy^iz \in A$  for each  $i \geq 0$ ,

(ii)  $|y| > 0$ , and

(iii)  $|xy| \leq p$ .

Prove that  $C = \{b^{2n}a^{2n}b^{3n} \mid n \geq 0\}$  is a nonregular language.