CIS 341, Spring 2006
Solutions for Midterm II

1. (a) True (Corollary 2.32)
   (b) False, e.g., $\{ 0^n1^n \mid n \geq 0 \}$ is context-free (lecture notes, page 2-5) but not regular (lecture notes, page 1-90).
   (c) False, e.g., $\{ 0^n1^n2^n \mid n \geq 0 \}$ is decidable (Homework 7, problem 2) but not context-free (lecture notes, page 2-105).
   (d) False by Theorem 3.16.
   (e) False, TM M can loop on string $w \notin L(M)$.
   (f) False. Consider DFA A that always rejects ($L(A) = \emptyset$; see lecture notes, page 1-19) and DFA B that always accepts ($L(B) = \Sigma^*$; see lecture notes, page 1-18). Then $L(A) \neq L(B)$ but $L(A) \cap \overline{L(B)} = \emptyset$. (However, it is true that two DFAs A and B are equivalent if and only if $(L(A) \cap \overline{L(B)}) \cup (\overline{L(A)} \cap L(B)) = \emptyset$; see Theorem 4.5.)
   (g) True by Theorem 3.21.
   (h) True by Theorem 4.2.
   (i) True by Homework 7, problem 3(b).
   (j) True by Homework 7, problem 3(a).

2. (a) A CFG $G = (V, \Sigma, R, S)$ is in Chomsky normal form means that each rule in R has one of the following forms:
   - $A \rightarrow BC$, where $A \in V$ and $B, C \in V - \{S\}$
   - $A \rightarrow x$, where $A \in V$ and $x \in \Sigma$
   - $S \rightarrow \varepsilon$ allowed.

   (b) A language L_1 that is Turing-recognizable has a Turing machine M_1 that may loop forever on a string $w \notin L_1$. A language L_2 that is Turing-decidable has a Turing machine M_2 that always halts.
   (c) The informal notion of an algorithm corresponds exactly to a Turing machine that always halts.
   (d) From Homework 5, problem 2(a).

$$S \rightarrow S \cup S \mid SS \mid S^* \mid (S) \mid 0 \mid 1 \mid e \mid \emptyset$$

3. This is from Homework 6, problem 2(a). Define the languages

$$\begin{aligned} A &= \{ a^m b^n c^n \mid m, n \geq 0 \} \text{ and} \\ B &= \{ a^n b^n c^m \mid m, n \geq 0 \} \end{aligned}$$

The language A is context free since it has CFG G_1 with rules

$$\begin{aligned} S &\rightarrow XY \\ X &\rightarrow aX \mid \varepsilon \\ Y &\rightarrow bYc \mid \varepsilon \end{aligned}$$

The language B is context free since it has CFG G_2 with rules

$$\begin{aligned} S &\rightarrow XY \\ X &\rightarrow aXb \mid \varepsilon \\ Y &\rightarrow aY \mid \varepsilon \end{aligned}$$

But $A \cap B = \{ a^n b^n c^n \mid n \geq 0\}$, which we know is not context free from Example 2.36 of the textbook.

4. This is basically Homework 7, problem 1.

   (a) $q_1 000 \quad \sqcup q_2 00 \quad \sqcup x q_3 0 \quad \sqcup x 0 q_4 \sqcup \quad \sqcup x 0 \sqcup q_{\text{reject}}$

   (b) $q_1 00 \quad \sqcup q_2 0 \quad \sqcup x q_3 \sqcup \quad \sqcup q_5 x \quad q_5 \sqcup x \quad \sqcup q_2 x \quad \sqcup x q_2 \sqcup \quad \sqcup x \sqcup q_{\text{accept}}$

5. This problem is a slight variation of Homework 8, problem 1. The equivalence problem for NFAs and regular expressions can be expressed as the language

   $$EQ_{\text{NFA,REX}} = \{ \langle N, R \rangle \mid N \text{ is an NFA and } R \text{ is a regular expression with } L(N) = L(R) \}.$$

   The following TM M shows that $EQ_{\text{NFA,REX}}$ is decidable:

   M = "On input $\langle N, R \rangle$, where N is an NFA and R is a regular expression:

   1. Check if $\langle N, R \rangle$ is a proper encoding. If not, *reject.*

   2. Convert N into an equivalent DFA D_1 using the algorithm in the theorem (1.39) that shows every NFA has an equivalent DFA.

   3. Convert R into an equivalent DFA D_2 by first using the algorithm in Kleene's theorem for converting a regular expression into an equivalent NFA (Lemma 1.55), and then converting the NFA into an equivalent DFA (Theorem 1.39).

   4. Run TM S on input $\langle D_1, D_2 \rangle$, where S is the TM that decides EQ_{DFA}, the language corresponding to the equivalence problem for DFAs, which is decidable (Theorem 4.5).

   5. If S accepts, *accept.* If S rejects, *reject.*