Midterm Exam II
CIS 341: Foundations of Computer Science II — **Spring 2006, day section**
Prof. Marvin K. Nakayama

Print family (or last) name: ____________________________________

Print given (or first) name: ____________________________________

I have read and understand all of the instructions below, and I will obey the Academic Honor Code.

Signature and Date: ____________________________________

- This exam has 6 pages in total, numbered 1 to 6. Make sure your exam has all the pages.
- This exam will be 1 hour and 25 minutes in length.
- This is a closed-book, closed-note exam.
- For all problems, follow these instructions:
  1. Give only your answers in the spaces provided. I will only grade what you put in the answer space, and I will take off points for any scratch work in the answer space. Use the scratch-work area or the backs of the sheets to work out your answers before filling in the answer space.
  2. DFA stands for deterministic finite automaton; NFA stands for nondeterministic finite automaton; CFG stands for context-free grammar; PDA stands for pushdown automaton.
  3. For any proofs, be sure to provide a step-by-step argument, with justifications for every step. Unless you are specifically asked to prove a theorem from the book, you may assume that the theorems in the textbook hold; i.e., you do not have to reprove the theorems in the textbook. When using a theorem from the textbook, make sure you provide enough detail so that it is clear which result you are using; e.g., say something like, "By the theorem that shows every NFA has an equivalent DFA, it follows that . . . "

Problem	1	2	3	4	5	Total
Points						

1. **[30 points]** For each of the following, circle TRUE if the statement is correct. Otherwise, circle FALSE

(a) TRUE FALSE — Every regular language is also context-free.

(b) TRUE FALSE — Every context-free language is also regular.

(c) TRUE FALSE — Every Turing-decidable language is also context-free.

(d) TRUE FALSE — There is a language A that is recognized by a nondeterministic Turing machine that is not recognized by any deterministic Turing machine.

(e) TRUE FALSE — For any Turing machine $M = (Q, \Sigma, \Gamma, \delta, q_0, q_{\text{accept}}, q_{\text{reject}})$ and any string $w \in \Sigma^*$, M will either accept or reject w.

(f) TRUE FALSE — Two DFAs A and B are equivalent if and only if $L(A) \cap \overline{L(B)} = \emptyset$.

(g) TRUE FALSE — A language A is Turing-recognizable if and only if some enumerator enumerates it.

(h) TRUE FALSE — The language

$$A_{\text{NFA}} = \{ \langle N, w \rangle \mid N \text{ is an NFA that accepts } w \}$$

is Turing-decidable.

(i) TRUE FALSE — The class of Turing-recognizable languages is closed under union.

(j) TRUE FALSE — The class of Turing-decidable languages is closed under union.

2. **[20 points]** Give a short answer (at most two sentences) for each part below. No proofs are required, but be sure to define any notation that you use.

   (a) What does it mean for a context-free grammar to be in Chomsky normal form?

   (b) What is the difference between Turing-recognizable and Turing-decidable.

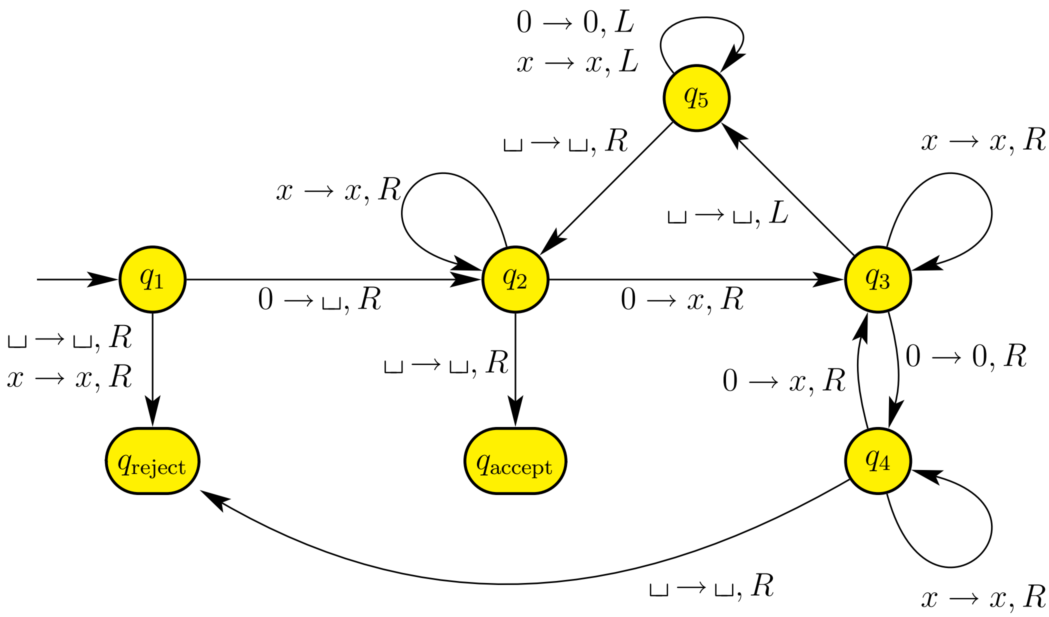
   (c) What is the Church-Turing Thesis?

   (d) Let $T = \{\, 0,\ 1,\ (,\ ),\ \cup,\ ^*,\ \emptyset,\ e \,\}$. We may think of T as the set of symbols used by regular expressions over the alphabet $\{0, 1\}$; the only difference is that we use e for symbol ε, to avoid potential confusion. Give a CFG G with set of terminals T that generates exactly the regular expressions with alphabet $\{0, 1\}$.

3. [**15 points**] Give an example of two context-free languages A and B whose intersection $C = A \cap B$ is not context-free. Explain your answer, and follow these instructions:

   - Provide CFGs for A and B.
   - If we showed in class or in a homework that your language C is not context-free, then you do not need to provide a proof of this fact; however, be sure to clearly state that your language C was proven to be non-context-free in class or in a homework.
   - If we did not show your language C is context-free in class or in a homework, then you will need to provide an explicit proof of this fact.

4. **[20 points]** The Turing machine M below recognizes the language $A = \{ 0^{2^n} \mid n \geq 0 \}$.



In each of the parts below, give the sequence of configurations that M enters when started on the indicated input string.

(a) 000

(b) 00

Scratch-work area

5. **[15 points]** Consider the problem of testing whether an NFA and a regular expression are equivalent. Express this problem as a language and show that it is decidable. For this problem, you can apply any theorem that we went over in class without proving it, but make sure that you give enough details so that it is clear what theorem you are using (e.g., say something like, "By the theorem that says the class of regular languages is closed under union, we can show that ....")