Midterm Exam
CS 341-451: Foundations of Computer Science II — **Fall 2007, eLearning section**
Prof. Marvin K. Nakayama

Print family (or last) name: ____

Print given (or first) name: ____

I have read and understand all of the instructions below, and I will obey the Academic Honor Code.

____
Signature and Date

- This exam has 8 pages in total, numbered 1 to 8. Make sure your exam has all the pages.
- Unless other arrangements have been made with the professor, the exam is to be given on Sunday, October 21, 2007. The exam is to last 2.5 hours.
- This is a closed-book, closed-note exam. No calculators are allowed.
- For all problems, follow these instructions:
  1. Give only your answers in the spaces provided. I will only grade what you put in the answer space, and I will take off points for any scratch work in the answer space. Use the scratch-work area or the backs of the sheets to work out your answers before filling in the answer space.
  2. DFA stands for deterministic finite automaton; NFA stands for nondeterministic finite automaton; CFG stands for context-free grammar; PDA stands for pushdown automaton.
  3. For any proofs, be sure to provide a step-by-step argument, with justifications for every step.

Problem	1	2	3	4	5	6	7	Total
Points								

1. **[20 points]** For each of the following, circle TRUE if the statement is correct. Otherwise, circle FALSE

(a) TRUE FALSE — The language $\{ a^n b^n a^n \mid n \geq 0 \}$ has regular expression $a^* b^* a^*$.

(b) TRUE FALSE — If $A = \{\varepsilon\}$ and $B = \{01, 00\}$, then $A \times B = A \circ B$.

(c) TRUE FALSE — Every regular language is also context-free.

(d) TRUE FALSE — Every context-free language is also regular.

(e) TRUE FALSE — Every context-free language is finite.

(f) TRUE FALSE — Every context-free language is infinite.

(g) TRUE FALSE — The class of regular languages is closed under concatenation.

(h) TRUE FALSE — If a language is regular, then it must have a regular expression.

(i) TRUE FALSE — If a language is recognized by a PDA, then the language has a CFG.

(j) TRUE FALSE — NFAs recognize nonregular languages.

2. **[20 points]** Give short answers to each of the following parts. **Each answer should be at most three sentences. Be sure to define any notation that you use.**

   (a) What does it mean for a list of strings to be in lexicographic order?

   (b) What does it mean for a context-free grammar $G = (V, \Sigma, R, S)$ to be in Chomsky normal form?

   (c) Give a regular expression for the language L consisting of strings over the alphabet $\Sigma = \{a, b\}$ that begin with a and end with b.

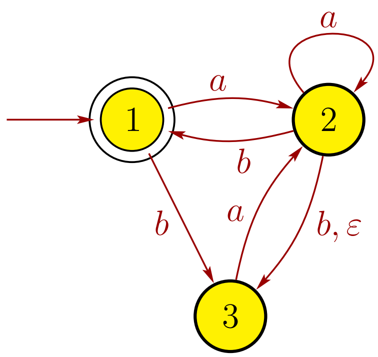
   (d) Give a DFA for the language consisting of strings over the alphabet $\Sigma = \{a, b\}$ that begin with a and end with b. You only need to draw the graph; do not specify the DFA as a 5-tuple.

3. [**10 points**] Let A be the language over the alphabet $\Sigma = \{a, b\}$ defined by regular expression $(b \cup baa)^* ab$. Give an NFA that recognizes A.

Draw an NFA for A here.

Scratch-work area

4. **[10 points]** Convert the following NFA into an equivalent DFA.



Answer:

Scratch-work area

5. **[20 points]** Let $\Sigma = \{a, b\}$, and consider the language $A = \{ w \in \Sigma^* \mid w = w^{\mathcal{R}} \}$, where $w^{\mathcal{R}}$ denotes the reverse of w.

   (a) Give a context-free grammar G that describes A. Be sure to specify G as a 4-tuple $G = (V, \Sigma, R, S)$.

   (b) Give a pushdown automaton that recognizes A. You only need to draw the picture.

Scratch-work area

6. **[10 points]** Is the class of context-free languages closed under intersection?

Circle one: YES NO

- If YES, give a proof.
- If NO, give an example of two context-free languages A and B whose intersection is not context-free. Also, give the rules of the context-free grammars for A and B. You do not need to prove that the intersection of A and B is not context-free.

7. **[10 points]** Recall the pumping lemma for regular languages:

Theorem: For every regular language L, there exists a pumping length p such that, if $s \in L$ with $|s| \geq p$, then we can write $s = xyz$ with

(i) $xy^iz \in L$ for each $i \geq 0$,

(ii) $|y| > 0$, and

(iii) $|xy| \leq p$.

Let $\Sigma = \{a, b\}$, and consider the language $A = \{\, w \in \Sigma^* \mid w = w^{\mathcal{R}} \}$, where $w^{\mathcal{R}}$ denotes the reverse of w. Prove that A is not a regular language.