Midterm Exam 1
CS 341: Foundations of Computer Science II — **Fall 2007, day section**
Prof. Marvin K. Nakayama

Print family (or last) name: ______________________________

Print given (or first) name: ______________________________

I have read and understand all of the instructions below, and I will obey the Academic Honor Code.

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Signature and Date

- This exam has 7 pages in total, numbered 1 to 7. Make sure your exam has all the pages.
- Note the number written on the upper right-hand corner of the first page. On the sign-up sheet being passed around, sign your name next to this number.
- This exam will be 1 hour and 25 minutes in length.
- This is a closed-book, closed-note exam.
- For all problems, follow these instructions:
  1. Give only your answers in the spaces provided. I will only grade what you put in the answer space, and I will take off points for any scratch work in the answer space. Use the scratch-work area or the backs of the sheets to work out your answers before filling in the answer space.
  2. For any proofs, be sure to provide a step-by-step argument, with justifications for every step.

Problem	1	2	3	4	5	Total
Points						

1. **[20 points]** For each of the following, circle TRUE if the statement is correct. Otherwise, circle FALSE

(a) TRUE FALSE — If a language A has an NFA, then A is nonregular.

(b) TRUE FALSE — The regular expressions $(a \cup b)^*$ and $(b^* a^*)^*$ generate the same language.

(c) TRUE FALSE — If a language A has a regular expression, then it also has a context-free grammar.

(d) TRUE FALSE — If a language A is recognized by a PDA, then it also is recognized by a DFA.

(e) TRUE FALSE — If a language A is recognized by a PDA, then it also is recognized by an NFA.

(f) TRUE FALSE — The class of context-free languages is closed under intersection.

(g) TRUE FALSE — The class of context-free languages is closed under complementation.

(h) TRUE FALSE — If A is a language generated by a context-free grammar in Chomsky normal form, then A must be regular.

(i) TRUE FALSE — If a language A is regular, then A^* must be regular.

(j) TRUE FALSE — If A_1 and A_2 are regular languages, then $A_1 \circ A_2$ must be context-free.

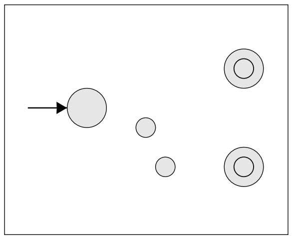
2. **[20 points]** Give short answers to each of the following parts. Each answer should be at most three sentences. Be sure to define any notation that you use.

   (a) What does it mean for a context-free grammar $G = (V, \Sigma, R, S)$ to be in Chomsky normal form?

   (b) Give an NFA with exactly four states for the language $\{ w \in \Sigma^* \mid w \text{ contains the substring } 110 \}$, where $\Sigma = \{0, 1\}$. You only need to draw the picture.

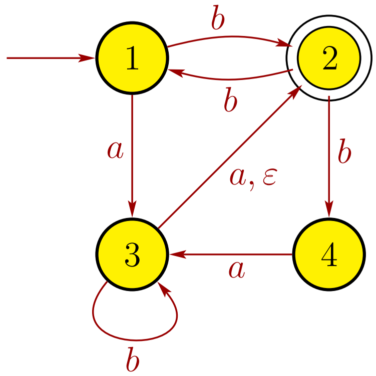
(c) Suppose that language A is recognized by NFA N_1 below. Note that the transitions are not drawn in N_1. Draw a picture of an NFA for A^*.

N_1



(d) Suppose that language A_1 has CFG $G_1 = (V_1, \Sigma, R_1, S_1)$ and language A_2 has CFG $G_2 = (V_2, \Sigma, R_2, S_2)$. Give a CFG G_3 for $A_1 \cup A_2$ in terms of G_1 and G_2. You do not have to prove the correctness of your CFG G_3, but do not give just an example.

3. **[20 points]** Let N be the following NFA with $\Sigma = \{a, b\}$, and let $C = L(N)$.



(a) List the strings in C in lexicographic order. If C has more than 5 strings, list only the first 5 strings in C, followed by 3 dots.

(b) Give a DFA for C.

Scratch-work area

4. **[25 points]** Consider the language

$$L = \{\, c^i a^j b^k \mid i, j, k \geq 0, \text{ and } i + j = k \,\}.$$

(a) Give a context-free grammar G for L. Be sure to specify G as a 4-tuple $G = (V, \Sigma, R, S)$.

(b) Give a PDA for L. You only need to draw the graph.

Scratch-work area

5. **[15 points]** Recall the pumping lemma for regular languages:

Theorem: If L is a regular language, then there exists a pumping length p where, if $s \in L$ with $|s| \geq p$, then there exists strings x, y, z such that $s = xyz$ and

(i) $xy^iz \in L$ for each $i \geq 0$,

(ii) $|y| \geq 1$, and

(iii) $|xy| \leq p$.

Consider the language $A = \{\, w \in \{a, b\}^* \mid w \text{ has more } a\text{'s than } b\text{'s} \,\}$. Specifically, for $w \in \{a, b\}^*$, let $n_a(w)$ be the number of a's in w, and let $n_b(w)$ be the number of b's in w. Then, $w \in A$ if and only if $n_a(w) > n_b(w)$. Prove that A is a nonregular language.