Midterm Exam 2
CS 341: Foundations of Computer Science II — **Fall 2007, day section**
Prof. Marvin K. Nakayama

Print family (or last) name: ____________________________________

Print given (or first) name: ____________________________________

I have read and understand all of the instructions below, and I will obey the Academic Honor Code.

Signature and Date: ____________________________________

- This exam has 7 pages in total, numbered 1 to 7. Make sure your exam has all the pages.
- This exam will be 1 hour and 25 minutes in length.
- This is a closed-book, closed-note exam.
- For all problems, follow these instructions:
  1. Give only your answers in the spaces provided. I will only grade what you put in the answer space, and I will take off points for any scratch work in the answer space. Use the scratch-work area or the backs of the sheets to work out your answers before filling in the answer space.
  2. DFA stands for deterministic finite automaton; NFA stands for nondeterministic finite automaton; CFG stands for context-free grammar; PDA stands for pushdown automaton.
  3. For any proofs, be sure to provide a step-by-step argument, with justifications for every step. Unless you are specifically asked to prove a theorem from the book, you may assume that the theorems in the textbook hold; i.e., you do not have to reprove the theorems in the textbook. When using a theorem from the textbook, make sure you provide enough detail so that it is clear which result you are using; e.g., say something like, "By the theorem that states $S^{**} = S^*$, it follows that ..."

Problem	1	2	3	4	5	6	Total
Points							

1. **[20 points]** For each of the following, circle TRUE if the statement is correct. Otherwise, circle FALSE

(a) TRUE FALSE — If A is a regular language, then A is Turing-recognizable.

(b) TRUE FALSE — It is impossible to define a correspondence between $\mathcal{N} = \{1, 2, 3, \ldots\}$ and $\mathcal{E} = \{2, 4, 6, \ldots\}$.

(c) TRUE FALSE — For any Turing machine $M = (Q, \Sigma, \Gamma, \delta, q_1, q_{\text{accept}}, q_{\text{reject}})$ and string $w \in \Sigma^*$, M will either accept or reject w.

(d) TRUE FALSE — Every Turing-decidable language is also context-free.

(e) TRUE FALSE — There is a Turing machine that decides if an arbitrary CFG generates ε.

(f) TRUE FALSE — DFAs A and B are equivalent if and only if $\overline{L(A)} \cap L(B) = \emptyset$.

(g) TRUE FALSE — The universal Turing machine U recognizes $\overline{A_{\text{TM}}}$, where $A_{\text{TM}} = \{ \langle M, w \rangle \mid M \text{ is a TM that accepts string } w \}$.

(h) TRUE FALSE — The language $E_{\text{DFA}} = \{ \langle D \rangle \mid D \text{ is a DFA with } L(D) = \emptyset \}$ is Turing-decidable.

(i) TRUE FALSE — For any alphabet Σ, Σ^* is countable.

(j) TRUE FALSE — There are some languages recognized by a 6-tape, nondeterministic Turing machine that cannot be recognized by a 1-tape, deterministic Turing machine.

2. **[20 points]** Give a short answer (at most three sentences) for each part below. Be sure to define any notation that you use.

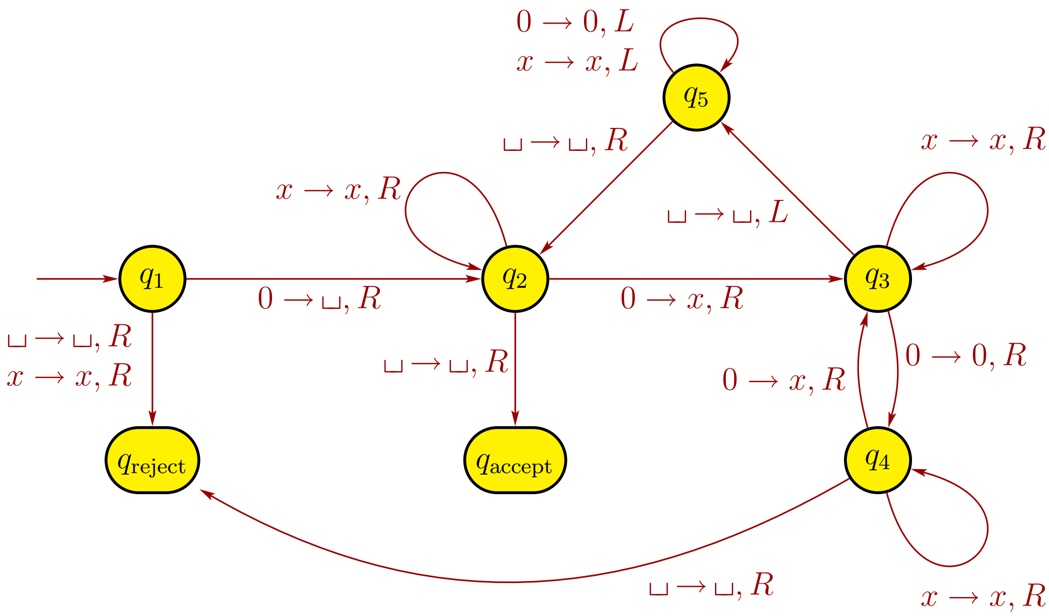
   (a) What is the difference between a Turing-recognizable language and a Turing-decidable language?

   (b) What does the Church-Turing Thesis say?

   (c) What does it mean for a function $f : A \to B$ to be one-to-one?

   (d) What does it mean for a function $f : A \to B$ to be onto?

3. **[20 points]** The Turing machine M below recognizes the language $A = \{\, 0^{2^n} \mid n \geq 0 \,\}$.



In each of the parts below, give the sequence of configurations that M enters when started on the indicated input string.

(a) 00

(b) 000

Scratch-work area

Each of the following problems requires you to prove a result. In your proofs, you can apply any theorems that we went over in class without proving them, except for the result you are asked to prove in the problem. When citing a theorem, make sure that you give enough details so that it is clear what theorem you are using (e.g., say something like, "By the theorem that says every context-free language has a CFG in Chomsky normal form, we can show that ....")

4. **[10 points]** Let $\mathcal{R}$ be the set of all real numbers. Show that $\mathcal{R}$ is uncountable.

5. **[15 points]** Consider the problem of determining **whether two regular expressions are equivalent**. Express this problem as a language and show that it is decidable.

6. **[15 points]** Recall that

$$
\begin{aligned}
E_{\mathrm{TM}} &= \{\, \langle M \rangle \mid M \text{ is a Turing machine with } L(M) = \emptyset \,\}, \\
EQ_{\mathrm{TM}} &= \{\, \langle M_1, M_2 \rangle \mid M_1 \text{ and } M_2 \text{ are Turing machines with } L(M_1) = L(M_2) \,\}.
\end{aligned}
$$

Prove that EQ_{TM} is undecidable by showing that E_{TM} reduces to EQ_{TM}. You may assume that E_{TM} is undecidable.