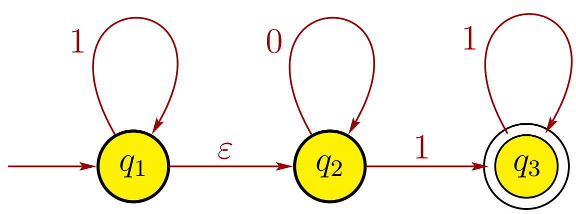
CS 341, Spring 2007
Solutions for Midterm 1

1. (a) False. Language a^* is infinite but regular.
   (b) False. a^*b^* generates $aaabb \notin \{a^n b^n | n \geq 0\}$.
   (c) True. If A has an NFA, then A is regular by Corollary 1.40. Corollary 2.32 implies that A is CFL, so A has a PDA by Lemma 2.21.
   (d) True. It has CFG $S \to 00S \mid 01S \mid 10S \mid 11S \mid \varepsilon$.
   (e) True. It has regular expression $((0 \cup 1)(0 \cup 1))^*$.
   (f) False. Homework 6, problem 2(a).
   (g) True. The class of languages recognized by DFAs is the class of regular languages, which is closed under complementation (Homework 2, problem 3).
   (h) False. Every CFL has a CFG in Chomsky normal form by Theorem 2.9. But not every CFL is regular, e.g., $\{a^n b^n | n \geq 0\}$.
   (i) False. $\{a^n b^n c^n | n \geq 0\}$ is neither regular nor context-free.
   (j) False. A nonregular language A cannot have a regular expression. If A had a regular expression, then Kleene's Theorem implies it must have a DFA, so A would be regular.

2. (a) Here is an NFA with exactly 3 states:



   (b) CFG $G = (V, \Sigma, R, S)$ is Chomsky normal form means that each rule in R has one of 3 forms:

$$
\begin{aligned}
A &\to BC \\
A &\to x \\
S &\to \varepsilon
\end{aligned}
$$

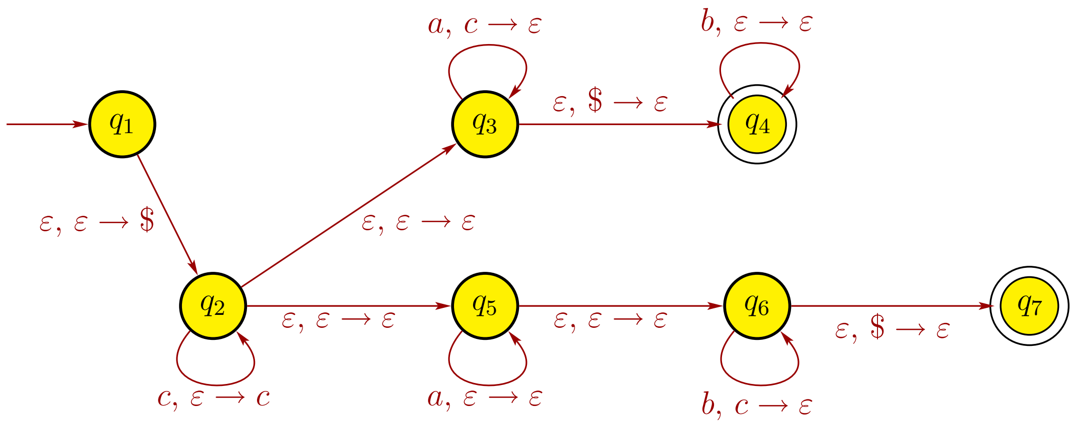
   where $A \in V$; $B, C \in V - \{S\}$; $x \in \Sigma$, and S is the start variable.
   (c) See page 1-50 of notes.
   (d) Homework 5, problem 3(c).

3. (a) $a, b, ab, ba, bb, aab, abb, bab, \ldots$
   (b) $(a \cup b^*(a \cup b))(b \cup a(b^*b \cup bb^*a))^*$

4. (a) $G = (V, \Sigma, R, S)$, where $V = \{S, W, X, Y, Z\}$, $\Sigma = \{a, b, c\}$, S is the start variable, and the rules R are

$$\begin{aligned} S &\rightarrow WX \mid Y \\ W &\rightarrow cWa \mid \varepsilon \\ X &\rightarrow bX \mid \varepsilon \\ Y &\rightarrow cYb \mid Z \\ Z &\rightarrow aZ \mid \varepsilon \end{aligned}$$

(b)



5. We prove this by contradiction. Suppose that A is a regular language. Let p be the "pumping length" of the Pumping Lemma. Consider the string $s = c^{3p}a^pb^{2p}$. Note that $s \in A$, and $|s| = 6p \geq p$, so the Pumping Lemma will hold. Thus, there exists strings x, y, and z such that $s = xyz$ and

   (a) $xy^iz \in A_3$ for each $i \geq 0$,
   (b) $|y| > 0$,
   (c) $|xy| \leq p$.

   Since the first p symbols of s are all c's, the third property implies that x and y consist only of c's. So z will be the rest of the c's, followed by a^pb^{2p}. The second property states that $|y| > 0$, so y has at least one c. More precisely, we can then say that

$$\begin{aligned} x &= c^j \text{ for some } j \geq 0, \\ y &= c^k \text{ for some } k \geq 1, \\ z &= c^m a^p b^{2p} \text{ for some } m \geq 0. \end{aligned}$$

   Since $c^{3p}a^pb^{2p} = s = xyz = c^jc^kc^ma^pb^{2p} = c^{j+k+m}a^pb^{2p}$, we must have that $j+k+m = 3p$. The first property implies that $xy^2z \in A$, but

$$\begin{aligned} xy^2z &= c^jc^kc^kc^ma^pb^{2p} \\ &= c^{3p+k}a^pb^{2p} \end{aligned}$$

since $j + k + m = 3p$. Hence, $xy^2z \notin A$ because $k \geq 1$, and we get a contradiction. Therefore, A is a nonregular language.