Midterm Exam 2
CS 341: Foundations of Computer Science II — **Spring 2007, day section**
Prof. Marvin K. Nakayama

Print family (or last) name: ____

Print given (or first) name: ____

I have read and understand all of the instructions below, and I will obey the Academic Honor Code.

Signature and Date: ____

- This exam has 7 pages in total, numbered 1 to 7. Make sure your exam has all the pages.
- This exam will be 1 hour and 25 minutes in length.
- This is a closed-book, closed-note exam.
- For all problems, follow these instructions:
  1. Give only your answers in the spaces provided. I will only grade what you put in the answer space, and I will take off points for any scratch work in the answer space. Use the scratch-work area or the backs of the sheets to work out your answers before filling in the answer space.
  2. DFA stands for deterministic finite automaton; NFA stands for nondeterministic finite automaton; CFG stands for context-free grammar; PDA stands for pushdown automaton.
  3. For any proofs, be sure to provide a step-by-step argument, with justifications for every step. Unless you are specifically asked to prove a theorem from the book, you may assume that the theorems in the textbook hold; i.e., you do not have to reprove the theorems in the textbook. When using a theorem from the textbook, make sure you provide enough detail so that it is clear which result you are using; e.g., say something like, "By the theorem that states $S^{**} = S^*$, it follows that ..."

Problem	1	2	3	4	5	Total
Points						

1. **[20 points]** For each of the following, circle TRUE if the statement is correct. Otherwise, circle FALSE

(a) TRUE FALSE — If A is regular, then A is Turing-recognizable.

(b) TRUE FALSE — If A is regular, then A is Turing-decidable.

(c) TRUE FALSE — If a language A is recognized by a nondeterministic Turing machine, then A is not Turing-recognizable.

(d) TRUE FALSE — Every 17-tape nondeterministic Turing machine has an equivalent 1-tape deterministic Turing machine.

(e) TRUE FALSE — If A is Turing-recognizable and Turing machine M recognizes A, then there must exist a string $w \notin A$ such that M loops on w.

(f) TRUE FALSE — There is a Turing machine that can take as input $\langle M, w \rangle$, where M is a Turing machine and w is a string, and decide if M accepts w.

(g) TRUE FALSE — We can use the universal Turing machine U to show that $\overline{A_{\text{TM}}}$ is Turing-recognizable.

(h) TRUE FALSE — The language $E_{\text{TM}} = \{ \langle M \rangle \mid M \text{ is a TM with } L(M) = \emptyset \}$ is decidable.

(i) TRUE FALSE — The language $E_{\text{DFA}} = \{ \langle D \rangle \mid D \text{ is a DFA with } L(D) = \emptyset \}$ is decidable.

(j) TRUE FALSE — The language $E_{\text{CFG}} = \{ \langle G \rangle \mid G \text{ is a CFG with } L(G) = \emptyset \}$ is decidable.

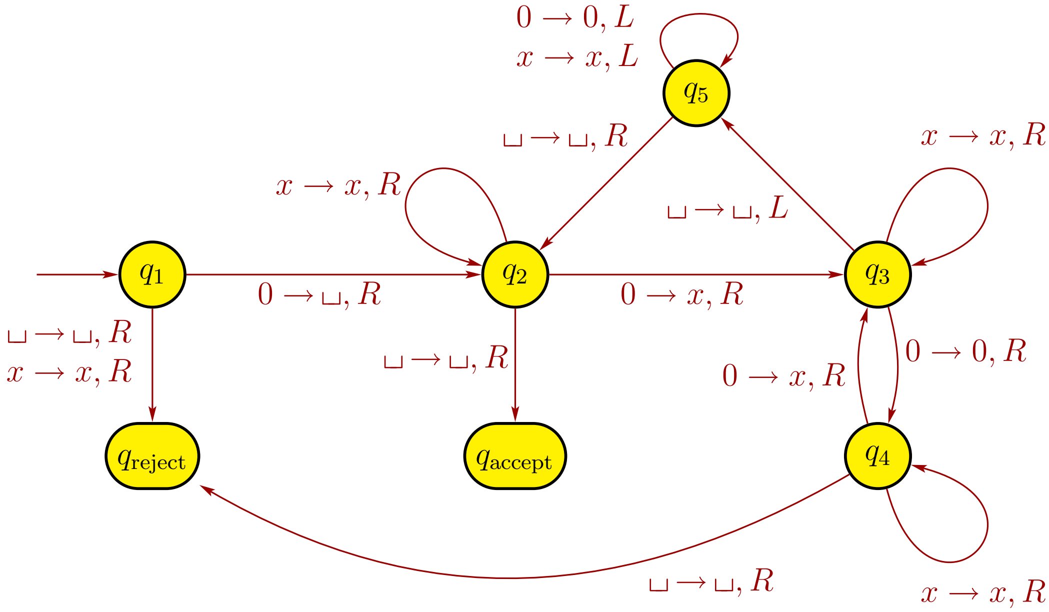
2. **[20 points]** Give a short answer (at most three sentences) for each part below. Be sure to define any notation that you use.

   (a) Explain the difference between Turing-recognizable and Turing-decidable.

   (b) What is the Church-Turing thesis?

3. [**10 points**] Let $\mathcal{B}$ be the set of infinite binary sequences. Show that $\mathcal{B}$ is uncountable.

4. **[20 points]** The Turing machine M below recognizes the language $A = \{ 0^{2^n} \mid n \geq 0 \}$.



In each of the parts below, give the sequence of configurations that M enters when started on the indicated input string.

(a) 00

(b) 000

Scratch-work area

5. **[15 points]** Consider the problem of testing whether a DFA and a regular expression are equivalent. Express this problem as a language and show that it is decidable. For this problem, you can apply any theorem that we went over in class without proving it, but make sure that you give enough details so that it is clear what theorem you are using (e.g., say something like, "By the theorem that says every context-free language has a CFG in Chomsky normal form, we can show that ....")

6. **[15 points]** Recall that

$$HALT_{\mathrm{TM}} = \{ \langle M, w \rangle \mid M \text{ is a Turing machine that halts on input } w \}.$$

Prove that $HALT_{\mathrm{TM}}$ is undecidable by showing that A_{TM} reduces to $HALT_{\mathrm{TM}}$, where

$$A_{\mathrm{TM}} = \{ \langle M, w \rangle \mid M \text{ is a Turing machine that accepts input } w \}.$$