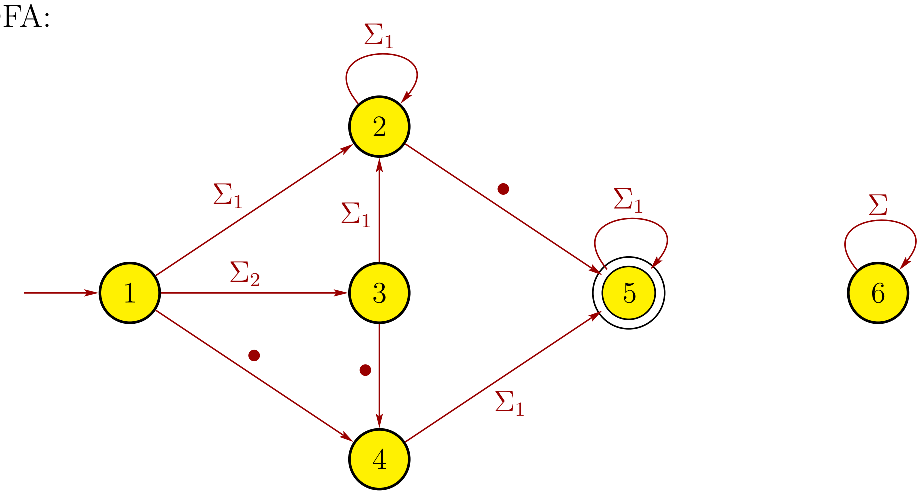
CS 341, Fall 2008
Solutions for Midterm, eLearning Section

1. (a) True. If A is recognized by an NFA, then A is regular by Corollary 1.40. But Corollary 2.32 then implies that A is also context-free.
   (b) True. Homework 4, problem 5a.
   (c) True. Homework 4, problem 5b.
   (d) True. Theorem 2.20.
   (e) True. Suppose that A is finite. Then slide 1-81 shows that A is regular, so A cannot be nonregular.
   (f) False. The language $A = \{0, 1\}^*$, which is infinite, has regular expression $(0 \cup 1)^*$. Thus, Theorem 1.54 implies A is regular.
   (g) False. The regular expression 0^*1^* generates the string $001 \notin \{0^n 1^n \mid n \geq 0\}$, so it cannot be a correct regular expression for the language. In fact, the language is not regular (slide 1-90), so it cannot have a regular expression by Theorem 1.54.
   (h) True. The language $\emptyset$ is finite, so slide 1-81 shows that it is regular. Corollary 2.32 then implies that $\emptyset$ is also context-free.
   (i) True. Homework 2, problem 3.
   (j) True. Homework 5, problem 3b.

2. (a) The set D is closed under f means that $x \in D$ implies $f(x) \in D$.
   (b) For a CFG $G = (V, \Sigma, R, S)$ to be in Chomsky normal form, each of its rules must have one of three forms: $A \to BC$, $A \to x$, or $S \to \varepsilon$, where A, B, C are variables, B and C are not the start variable, x is a terminal, and S is the start variable.

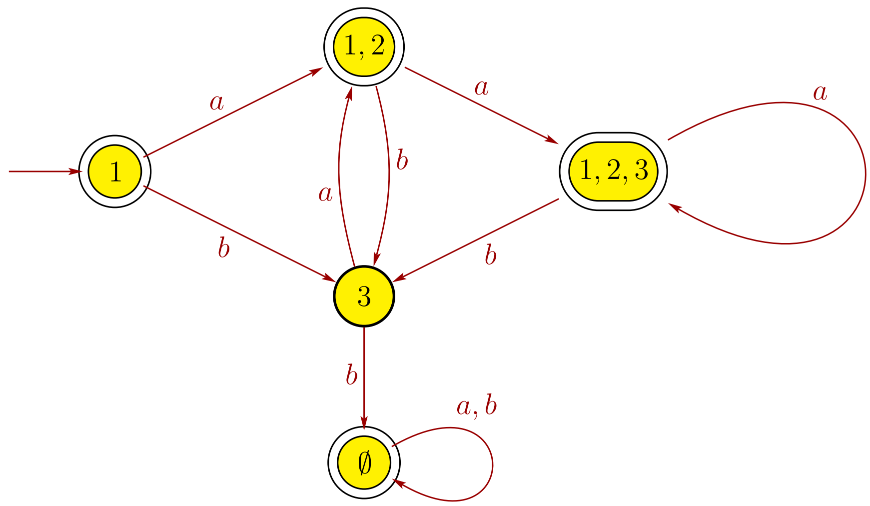
3. (a) Regular expression: $(+ \cup - \cup \varepsilon)(\Sigma_1 \Sigma_1^* . \Sigma_1^* \ \cup \ .\Sigma_1 \Sigma_1^*)$
   (b) DFA:



All transitions not specified go to state 6.

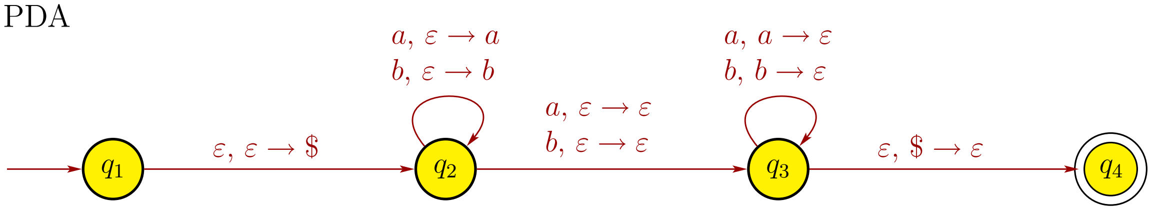
4. Homework 6, problem 2a.

5. DFA



6. (a) $G = (V, \Sigma, R, S)$, where $V = \{S\}$, $\Sigma = \{a, b\}$, and the rules are $S \to aSa \mid bSb \mid a \mid b$.

(b) PDA



7. Suppose that $A = \{c^{2n}a^nb^{2n} \mid n \geq 0\}$ is a regular language. Let p be the pumping length, and consider the string $s = c^{2p}a^pb^{2p} \in A$. Note that $|s| = 5p \geq p$, so the pumping lemma implies we can write $s = xyz$ with $xy^iz \in A$ for all $i \geq 0$, $|y| > 0$, and $|xy| \leq p$. Now, $|xy| \leq p$ implies that x and y have only c's (together up to p in total) and z has the rest of the c's, followed by a^pb^{2p}. Hence, we can write $x = c^j$ for some $j \geq 0$, $y = c^k$ for some $k \geq 0$, and $z = c^\ell a^pb^{2p}$, where $j + k + \ell = 2p$ since $xyz = s = c^{2p}a^pb^{2p}$. Also, $|y| > 0$ implies $k > 0$. Now consider the string $xyyz = c^jc^kc^kc^\ell a^pb^{2p} = c^{2p+k}a^pb^{2p}$ since $j + k + \ell = 2p$. Note that $xyyz \notin A$ since $k > 0$, which contradicts (i), so A is not a regular language.