Midterm Exam 1
CS 341: Foundations of Computer Science II — **Spring 2008, day section**
Prof. Marvin K. Nakayama

Print family (or last) name: ______________________________

Print given (or first) name: ______________________________

I have read and understand all of the instructions below, and I will obey the Academic Honor Code.

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Signature and Date

- This exam has 7 pages in total, numbered 1 to 7. Make sure your exam has all the pages.
- Note the number written on the upper right-hand corner of the first page. On the sign-up sheet being passed around, sign your name next to this number.
- This exam will be 1 hour and 25 minutes in length.
- This is a closed-book, closed-note exam.
- For all problems, follow these instructions:
  1. Give only your answers in the spaces provided. I will only grade what you put in the answer space, and I will take off points for any scratch work in the answer space. Use the scratch-work area or the backs of the sheets to work out your answers before filling in the answer space.
  2. For any proofs, be sure to provide a step-by-step argument, with justifications for every step.

Problem	1	2	3	4	5	Total
Points						

1. **[20 points]** For each of the following, circle TRUE if the statement is correct. Otherwise, circle FALSE

(a) TRUE FALSE — If a language A is nonregular, then it has an NFA.

(b) TRUE FALSE — The language $\{ 0^n 1^n \mid n \geq 0\}$ has regular expression $\varepsilon \cup 01 \cup 0011 \cup 000111 \cup \cdots$.

(c) TRUE FALSE — If a language is regular, then it must be finite.

(d) TRUE FALSE — If a language A is recognized by a DFA, then it must also be recognized by a PDA.

(e) TRUE FALSE — If A is any language, then A must have a PDA.

(f) TRUE FALSE — The class of context-free languages is closed under intersection.

(g) TRUE FALSE — The class of context-free languages is closed under concatenation.

(h) TRUE FALSE — If a language A has a regular expression, then it must also have an DFA.

(i) TRUE FALSE — If language A is regular, then $\overline{A}$ must be regular.

(j) TRUE FALSE — $\emptyset$ is a regular language.

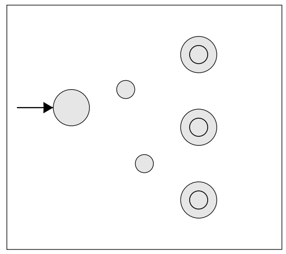
2. **[20 points]** Give short answers to each of the following parts. Each answer should be at most three sentences. Be sure to define any notation that you use.

   (a) Give an NFA with exactly four states for the language $\{\, w \in \Sigma^* \mid w \text{ contains the substring } 001 \,\}$, where $\Sigma = \{0, 1\}$. You only need to draw the picture.

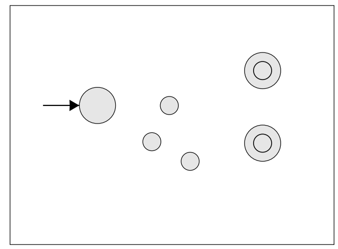
   (b) What does it mean for a context-free grammar $G = (V, \Sigma, R, S)$ to be in Chomsky normal form?

(c) Suppose that language A_1 is recognized by NFA N_1 below, and language A_2 is recognized by NFA N_2 below. Note that the transitions are not drawn in N_1 and N_2. Draw a picture of an NFA for $A_1 \circ A_2$.

N_1

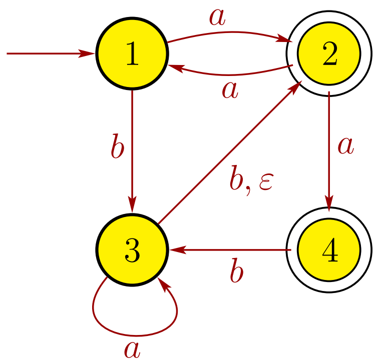


N_2



(d) Suppose that language A_1 has CFG $G_1 = (V_1, \Sigma, R_1, S_1)$. Give a CFG G_2 for A_1^*. You do not have to prove the correctness of your CFG G_2, but do not give just an example.

3. **[20 points]** Let N be the following NFA with $\Sigma = \{a, b\}$, and let $C = L(N)$.



(a) List the strings in C in lexicographic order. If C has more than 5 strings, list only the first 5 strings in C, followed by 3 dots.

(b) Give a regular expression for C.

Scratch-work area

4. **[25 points]** Consider the language

$$L = \{\, c^i a^j b^k \mid i, j, k \geq 0, \text{ and } i = j \text{ or } i = k \,\}.$$

(a) Give a context-free grammar G for L. Be sure to specify G as a 4-tuple $G = (V, \Sigma, R, S)$.

(b) Give a PDA for L. You only need to draw the graph.

Scratch-work area

5. **[15 points]** Recall the pumping lemma for regular languages:

Theorem: If L is a regular language, then there exists a pumping length p where, if $s \in L$ with $|s| \geq p$, then there exists strings x, y, z such that $s = xyz$ and (i) $xy^iz \in L$ for each $i \geq 0$, (ii) $|y| \geq 1$, and (iii) $|xy| \leq p$.

Consider the context-free grammar $G = (V, \Sigma, R, S)$, with $V = \{S\}$, $\Sigma = \{+, -, \times, /, (,), 0, 1, \ldots, 9\}$ and rules R given by

$$S \rightarrow S + S \mid S - S \mid S \times S \mid S/S \mid (S) \mid 0 \mid 1 \mid 2 \mid \cdots \mid 9$$

Let $A = L(G)$. Is A a regular or nonregular language? If A is regular, give a regular expression for A. If A is not regular, prove that it is a nonregular language.

Circle one: **Regular Language** **Nonregular Language**