CS 341, Spring 2008
Solutions for Midterm 2

1. (a) True, by Theorem 4.9.
   (b) True, by Theorem 4.9.
   (c) False, e.g., $\overline{A_{\text{TM}}}$ is not Turing-recognizable.
   (d) False. A TM M may loop on input w.
   (e) True. List the strings in lexicographic order.
   (f) False. Homework 9, problem 1.
   (g) True, by Theorems 3.13 and 3.16.
   (h) False, by Theorem 5.4.
   (i) True, by Theorem 4.5.
   (j) False, by Corollary 4.23.

2. (a) Function f is one-to-one means that if $x \neq y$, then $f(x) \neq f(y)$.
   (b) A language L_1 that is Turing-recognizable has a Turing machine M_1 that may loop forever on a string $w \notin L_1$. A language L_2 that is Turing-decidable has a Turing machine M_2 that always halts.

3. (a) q_1aa $\quad \sqcup q_2a$ $\quad \sqcup xq_3\sqcup$ $\quad \sqcup q_5x$ $\quad q_5\sqcup x$ $\quad \sqcup q_2x$ $\quad \sqcup xq_2\sqcup$ $\quad \sqcup x\sqcup q_{\text{accept}}$

   (b) q_1aaaaa $\quad \sqcup q_2aaaa$ $\quad \sqcup xq_3aaa$ $\quad \sqcup xaq_4aa$
   $\sqcup xaxq_3a$ $\quad \sqcup xaxaq_4$ $\quad \sqcup xaxa\sqcup q_{\text{reject}}$

4. See slides 4-39 and 4-40.

5. Define the language as

$$C = \{\langle M, N\rangle \mid M \text{ and } N \text{ are NFAs with } L(M) = L(N)\,\}.$$

Recall that the proof of Theorem 4.5 defines a Turing machine F that decides the language $EQ_{\text{DFA}} = \{\,\langle A, B\rangle \mid A \text{ and } B \text{ are DFAs and } L(A) = L(B)\,\}$. Then the following Turing machine T decides C:

T = "On input $\langle M, N\rangle$, where M and N are DFAs:
  1. Convert M and N into equivalent DFAs D_1 and D_2 using the algorithm in the proof of Kleene's Theorem.
  2. Run TM F from Theorem 4.5 on input $\langle D_1, D_2\rangle$.
  3. If F accepts, *accept*. If F rejects, *reject*."

6. This is Theorem 5.1, whose proof is given on slide 5-8.