Midterm Exam 2
CS 341: Foundations of Computer Science II — **Spring 2008, day section**
Prof. Marvin K. Nakayama

Print family (or last) name: ____________________________________________

Print given (or first) name: ____________________________________________

I have read and understand all of the instructions below, and I will obey the Academic Honor Code.

Signature and Date: ____________________________________________

- This exam has 7 pages in total, numbered 1 to 7. Make sure your exam has all the pages.
- This exam will be 1 hour and 25 minutes in length.
- This is a closed-book, closed-note exam.
- For all problems, follow these instructions:
  1. Give only your answers in the spaces provided. I will only grade what you put in the answer space, and I will take off points for any scratch work in the answer space. Use the scratch-work area or the backs of the sheets to work out your answers before filling in the answer space.
  2. DFA stands for deterministic finite automaton; NFA stands for nondeterministic finite automaton; CFG stands for context-free grammar; PDA stands for pushdown automaton.
  3. For any proofs, be sure to provide a step-by-step argument, with justifications for every step. Unless you are specifically asked to prove a theorem from the book, you may assume that the theorems in the textbook hold; i.e., you do not have to reprove the theorems in the textbook. When using a theorem from the textbook, make sure you provide enough detail so that it is clear which result you are using; e.g., say something like, "By the theorem that states $S^{**} = S^*$, it follows that ..."

| Problem | 1 | 2 | 3 | 4 | 5 | 6 | Total |
|---|---|---|---|---|---|---|---|
| Points | | | | | | | |

1. **[20 points]** For each of the following, circle TRUE if the statement is correct. Otherwise, circle FALSE

(a) TRUE FALSE — If $A$ is context-free, then $A$ is Turing-recognizable.

(b) TRUE FALSE — If $A$ is context-free, then $A$ is Turing-decidable.

(c) TRUE FALSE — Every language is Turing-recognizable.

(d) TRUE FALSE — For a Turing machine $M$ and a string $w$, $M$ either accepts or rejects $w$.

(e) TRUE FALSE — The language $(0 \cup 1)^*$ is countable.

(f) TRUE FALSE — The set $\mathcal{B}$ of infinite binary sequences is countable.

(g) TRUE FALSE — If language $A$ is recognized by a 14-tape nondeterministic Turing machine, then there is a single-tape deterministic Turing machine that also recognizes $A$.

(h) TRUE FALSE — The language $EQ_{\text{TM}}$ is decidable, where

$$EQ_{\text{TM}} = \{ \langle M, N \rangle \mid M \text{ and } N \text{ are TMs with } L(M) = L(N) \}.$$

(i) TRUE FALSE — The language $EQ_{\text{DFA}}$ is decidable, where

$$EQ_{\text{DFA}} = \{ \langle C, D \rangle \mid C \text{ and } D \text{ are DFAs with } L(C) = L(D) \}.$$

(j) TRUE FALSE — The language $\overline{A_{\text{TM}}}$ is decidable, where

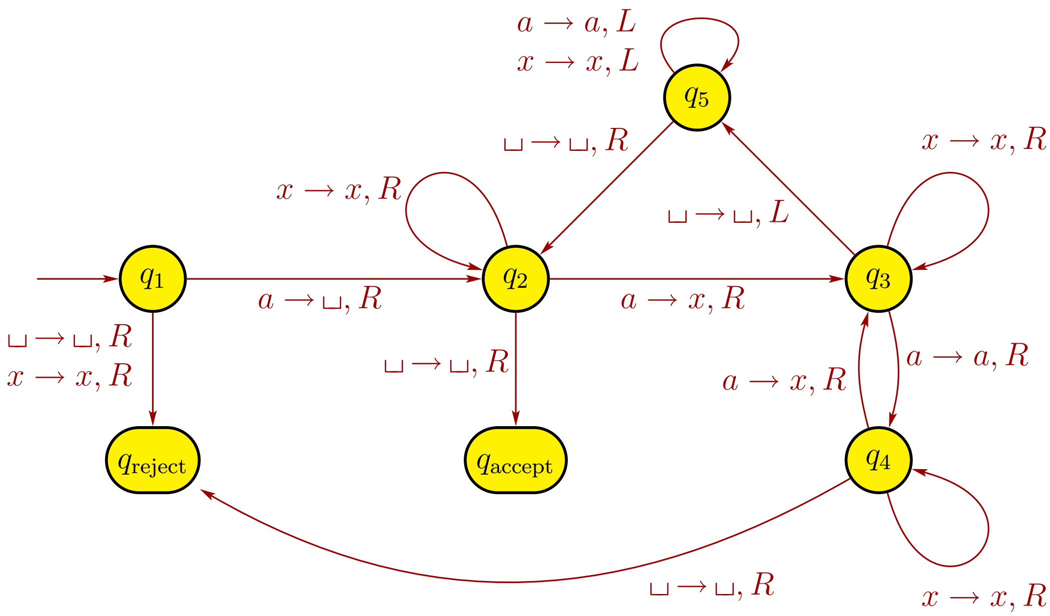
$$A_{\text{TM}} = \{ \langle M, w \rangle \mid M \text{ is a TM that accepts input } w \}.$$

2. **[15 points]** Give a short answer (at most three sentences) for each part below. Be sure to define any notation that you use.

   (a) What does it mean for a function $f : A \rightarrow B$ to be one-to-one?

   (b) Explain the difference between Turing-recognizable and Turing-decidable.

3. **[20 points]** The Turing machine $M$ below recognizes the language $A = \{ a^{2^n} \mid n \geq 0 \}$.




In each of the parts below, give the sequence of configurations that $M$ enters when started on the indicated input string.

(a) $aa$

(b) $aaaaa$

**Scratch-work area**

Each of the following problems requires you to prove a result. In your proofs, you can apply any theorems that we went over in class without proving them, except for the result you are asked to prove in the problem. When citing a theorem, make sure that you give enough details so that it is clear what theorem you are using (e.g., say something like, "By the theorem that says every context-free language has a CFG in Chomsky normal form, we can show that ....")

4. [**15 points**] Let $\mathcal{L}$ be the set of all languages having alphabet $\Sigma = \{0, 1\}$. Show that $\mathcal{L}$ is uncountable.

5. [**15 points**] Consider the decision problem of testing whether two NFAs are equivalent. Express this problem as a language and show that it is decidable.

6. **[15 points]** Recall that

$$HALT_{\text{TM}} = \{ \langle M, w \rangle \mid M \text{ is a Turing machine that halts on input } w \}.$$

Prove that $HALT_{\text{TM}}$ is undecidable by showing that $A_{\text{TM}}$ reduces to $HALT_{\text{TM}}$, where

$$A_{\text{TM}} = \{ \langle M, w \rangle \mid M \text{ is a Turing machine that accepts input } w \}.$$