1. (a) True. The language \( \emptyset \) is finite, so slide 1-81 shows that it is regular. Corollary 2.32 then implies that \( \emptyset \) is also context-free.

(b) True. Suppose \( A \) is finite. Then \( A \) is regular by slide 1-81, so \( A \) can’t be nonregular.

(c) True. Theorem 2.9.

(d) False. Homework 5, problem 1(a), is a context-free language that is also regular since it has regular expression \( 0^*1^*0^*1^*(0 \cup 1)^* \).

(e) True. If \( B \) is regular, then so is \( \overline{B} \) by Homework 2, problem 3. Then \( A \cap \overline{B} \) is regular by slide 1-28.

(f) False. The language \( A = \{0,1\}^* \), which is infinite, has regular expression \( (0 \cup 1)^* \). Thus, Theorem 1.54 implies \( A \) is regular.

(g) False. The regular expression \( 1^*0^*1^* \) generates the string \( 001 \not\in \{1^n0^n1^n \mid n \geq 0 \} \), so it cannot be a correct regular expression for the language. In fact, the language is nonregular, so it cannot have a regular expression.

(h) False. The derivation \( S \Rightarrow 0 \) generates the string 0, which is not in the language, so the CFG cannot be correct.


(j) False. The language \( A = \{0^n1^n \mid n \geq 0 \} \) is nonregular, and \( B = \emptyset \) is a subset of \( A \), but \( B \) is regular since it is finite.

2. (a) Shorter strings appear before longer strings, and strings of the same length are in alphabetical order.

(b) \( S \rightarrow Ya \) is not in Chomsky normal form since the CFG cannot have a right-hand side (RHS) that is a mix of terminals and variables.

\( X \rightarrow YS \) is improper since \( S \) cannot be on the RHS of a rule.

\( Y \rightarrow \varepsilon \) is improper since \( \varepsilon \) cannot be on the RHS of rule when the left side is not \( S \).

\( Y \rightarrow YXY \) is improper since the RHS cannot have more than two variables.

(c) slide 1-53.

(d) The set \( D \) is closed under \( f \) means that \( x \in D \) implies \( f(x) \in D \).

3. (a) Regular expression: \( (+ \cup - \cup \varepsilon)(\Sigma_1 \Sigma_1^* \Sigma_1^* \cup \Sigma_1 \Sigma_1^*) \)

(b) DFA:
All transitions not specified go to state 6.


5. DFA

6. $G = (V, \Sigma, R, S)$, with $V = \{S, X\}$, $\Sigma = \{a, b, c\}$, start variable $S$ and rules $S \rightarrow cSb \mid X$ and $X \rightarrow aXb \mid \varepsilon$.

7. Suppose that $A$ is a regular language. Let $p$ be the pumping length, and consider the string $s = a^pbba^p \in A$. Note that $|s| = 2p + 2 \geq p$, so the pumping lemma implies we can write $s = xyz$ with $xy^iz \in A$ for all $i \geq 0$, $|y| > 0$, and $|xy| \leq p$. Now, $|xy| \leq p$ implies that $x$ and $y$ have only $a$’s (together up to $p$ in total) and $z$ has the rest of the $a$’s at the beginning, followed by $bba^p$. Hence, we can write $x = a^j$ for some $j \geq 0$, $y = a^k$ for some $k \geq 0$, and $z = a^\ell bba^p$, where $j + k + \ell = p$ since $xyz = s = a^pbba^p$. Also, $|y| > 0$ implies $k > 0$. Now consider the string $xyyz = a^j a^k a^\ell bba^p = a^{p+k} bba^p$ since $j + k + \ell = p$. Note that $xyyz \notin A$ since it is not the same forwards and backwards because $k > 0$, which contradicts (i), so $A$ is not a regular language.