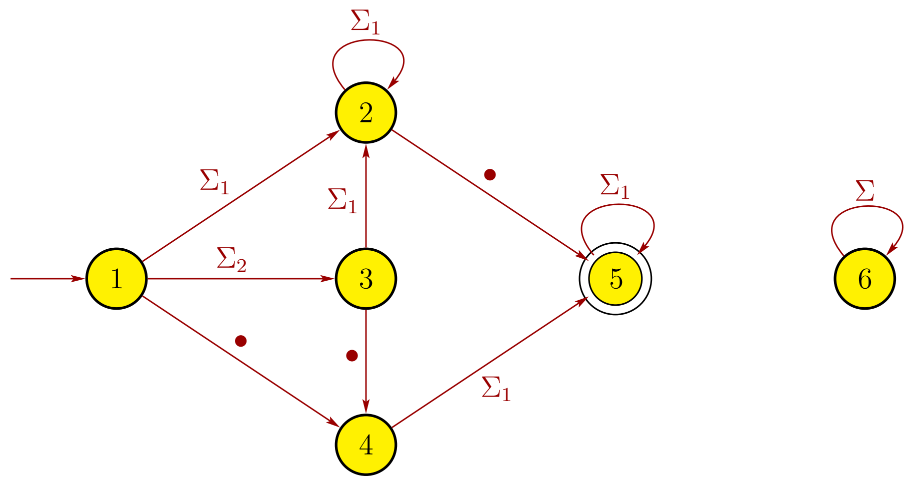
CS 341, Fall 2009
Solutions for Midterm, eLearning Section

1. (a) True. The language $\emptyset$ is finite, so slide 1-81 shows that it is regular. Corollary 2.32 then implies that $\emptyset$ is also context-free.
   (b) True. Suppose A is finite. Then A is regular by slide 1-81, so A can't be nonregular.
   (c) True. Theorem 2.9.
   (d) False. Homework 5, problem 1(a), is a context-free language that is also regular since it has regular expression $0^*1^*0^*1^*0^*1^*(0 \cup 1)^*$.
   (e) True. If B is regular, then so is $\overline{B}$ by Homework 2, problem 3. Then $A \cap \overline{B}$ is regular by slide 1-28.
   (f) False. The language $A = \{0,1\}^*$, which is infinite, has regular expression $(0 \cup 1)^*$. Thus, Theorem 1.54 implies A is regular.
   (g) False. The regular expression $1^*0^*1^*$ generates the string $001 \notin \{1^n0^n1^n \mid n \geq 0\}$, so it cannot be a correct regular expression for the language. In fact, the language is nonregular, so it cannot have a regular expression.
   (h) False. The derivation $S \Rightarrow 0$ generates the string 0, which is not in the language, so the CFG cannot be correct.
   (i) True. Slide 1-28.
   (j) False. The language $A = \{0^n1^n | n \geq 0\}$ is nonregular, and $B = \emptyset$ is a subset of A, but B is regular since it is finite.

2. (a) Shorter strings appear before longer strings, and strings of the same length are in alphabetical order.
   (b)
   - $S \to Ya$ is not in Chomsky normal form since the CFG cannot have a right-hand side (RHS) that is a mix of terminals and variables.
   - $X \to YS$ is improper since S cannot be on the RHS of a rule.
   - $Y \to \varepsilon$ is improper since ε cannot be on the RHS of rule when the left side is not S.
   - $Y \to YXY$ is improper since the RHS cannot have more than two variables.

   (c) slide 1-53.
   (d) The set D is closed under f means that $x \in D$ implies $f(x) \in D$.

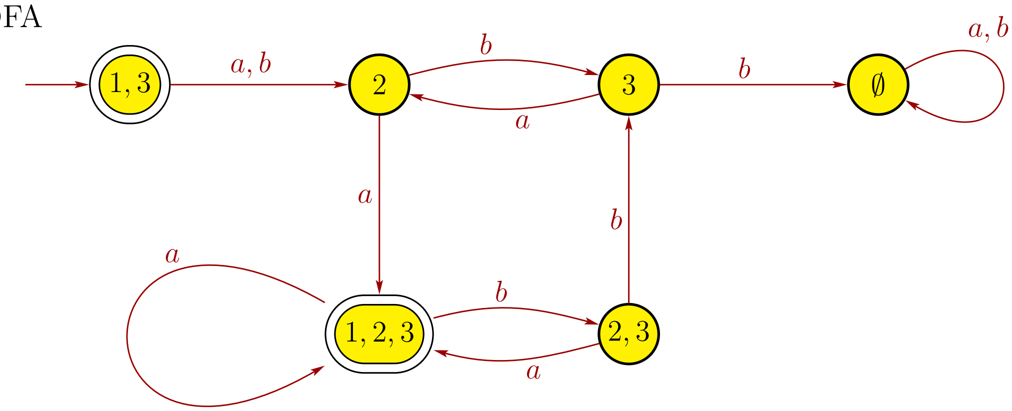
3. (a) Regular expression: $(+ \cup - \cup \varepsilon)(\Sigma_1\Sigma_1^*.\Sigma_1^* \ \cup \ .\Sigma_1\Sigma_1^*)$
   (b) DFA:



All transitions not specified go to state 6.

4. Homework 3, problem 2.

5. DFA



6. $G = (V, \Sigma, R, S)$, with $V = \{S, X\}$, $\Sigma = \{a, b, c\}$, start variable S and rules $S \to cSb \mid X$ and $X \to aXb \mid \varepsilon$.

7. Suppose that A is a regular language. Let p be the pumping length, and consider the string $s = a^p bba^p \in A$. Note that $|s| = 2p + 2 \geq p$, so the pumping lemma implies we can write $s = xyz$ with $xy^iz \in A$ for all $i \geq 0$, $|y| > 0$, and $|xy| \leq p$. Now, $|xy| \leq p$ implies that x and y have only a's (together up to p in total) and z has the rest of the a's at the beginning, followed by bba^p. Hence, we can write $x = a^j$ for some $j \geq 0$, $y = a^k$ for some $k \geq 0$, and $z = a^\ell bba^p$, where $j + k + \ell = p$ since $xyz = s = a^p bba^p$. Also, $|y| > 0$ implies $k > 0$. Now consider the string $xyyz = a^j a^k a^k a^\ell bba^p = a^{p+k}bba^p$ since $j + k + \ell = p$. Note that $xyyz \notin A$ since it is not the same forwards and backwards because $k > 0$, which contradicts (i), so A is not a regular language.