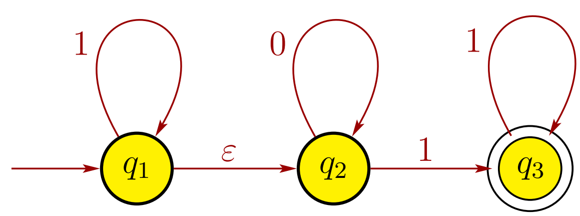
CS 341, Spring 2009
Solutions for Midterm 1

1. (a) True. Finite languages are regular by the theorem on slide 1-81.
   (b) False. The language $A = \{0^n 1^n \mid n \geq 0\}$ is context-free since it has CFG with rules $S \to 0S1 \mid \varepsilon$, but it is non-regular, as shown on slide 1-90.
   (c) False. The language a^* is regular but infinite.
   (d) False. Corollary 1.40 shows that a language is regular if and only if it has an NFA.
   (e) False. Homework 6, problem 2(a).
   (f) True. If A has a regular expression, then it must be regular by Theorem 1.54. Homework 2, problem 3, shows that $\overline{A}$ must be regular. Corollary 2.32 implies that $\overline{A}$ must be context-free.
   (g) True. By Corollary 1.40 and Theorem 1.54.
   (h) False. Every string generated by the given regular expression must end in 0, so the string 01 cannot be generated by the regular expression, even though it has an odd number of 0's.
   (i) True, by Lemma 2.27 and Theorem 2.9.
   (j) False. Let $A = \{0,1\}^*$ and $B = \{ww \mid w \in \{0,1\}^*\}$. Language A has CFG with rules $S \to 0S \mid 1S \mid \varepsilon$, so it is context-free. Slide 2-108 shows that B is not context-free.

2. (a) Here is an NFA with exactly 3 states:



   (b) CFG $G = (V, \Sigma, R, S)$ is Chomsky normal form means that each rule in R has one of 3 forms:

$$\begin{aligned} A &\to BC \\ A &\to x \\ S &\to \varepsilon \end{aligned}$$

where $A \in V$; $B, C \in V - \{S\}$; $x \in \Sigma$, and S is the start variable. Thus, the violating rules are

- $S \to aX$
- $X \to YXY$

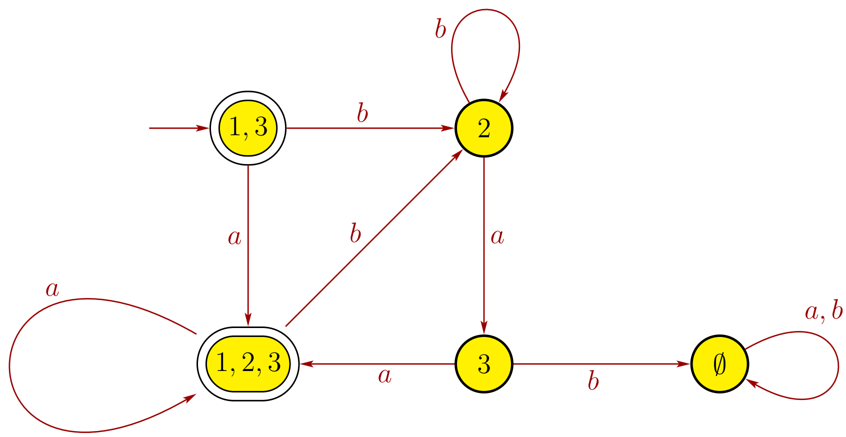
- $X \to \varepsilon$
- $Y \to XS$ (since S cannot be on the right-side of a rule)

(c) See page 1-53 of notes.

(d) Homework 5, problem 3(b).

3. (a) $\varepsilon, a, aa, aaa, baa, \ldots$

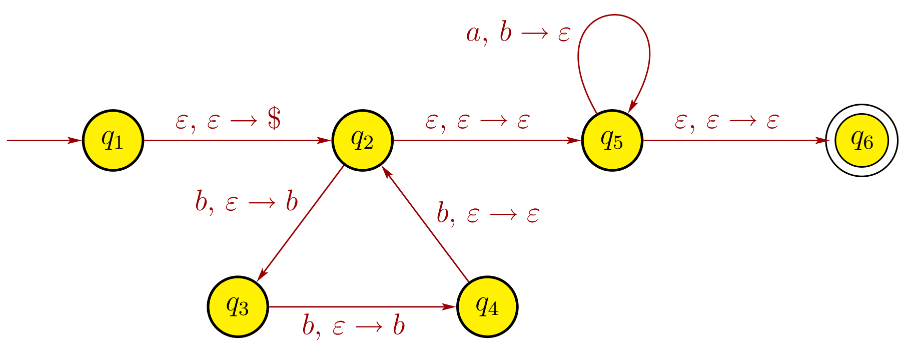
   (b) A DFA for C is below:



4. (a) $G = (V, \Sigma, R, S)$ with set of variables $V = \{S\}$, where S is the start variable; set of terminals $\Sigma = \{a, b\}$; and rules

$$S \;\to\; bbbSaa \mid \varepsilon$$

   (b)



5. Language A is nonregular. We prove this by contradiction. Suppose that A is a regular language. Let p be the "pumping length" of the Pumping Lemma. Consider the string $s = a^{2p}c^{3p}b^{2p}$. Note that $s \in A$, and $|s| = 7p > p$, so the Pumping Lemma will hold. Thus, there exists strings x, y, and z such that $s = xyz$ and

   (a) $xy^iz \in A$ for each $i \geq 0$,

   (b) $|y| > 0$,

   (c) $|xy| \leq p$.

Since the first p symbols of s are all a's, the third property implies that x and y consist only of a's. So z will be the rest of the a's, followed by $c^{3p}b^{2p}$. The second property states that $|y| > 0$, so y has at least one a. More precisely, we can then say that

$$\begin{aligned} x &= a^j \text{ for some } j \geq 0, \\ y &= a^k \text{ for some } k \geq 1, \\ z &= a^m c^{3p} b^{2p} \text{ for some } m \geq 0. \end{aligned}$$

Since $a^{2p}c^{3p}b^{2p} = s = xyz = a^j a^k a^m c^{3p} b^{2p} = a^{j+k+m} c^{3p} b^{2p}$, we must have that

$$j + k + m = 2p.$$

The first property implies that $xy^2z \in A$, but

$$\begin{aligned} xy^2z &= a^j a^k a^k a^m c^{3p} b^{2p} \\ &= a^{2p+k} c^{3p} b^{2p} \end{aligned}$$

since $j + k + m = 2p$. Hence, $xy^2z \notin A$ since $k \geq 1$, and we get a contradiction. Therefore, A is a nonregular language.