Midterm Exam 1
CS 341: Foundations of Computer Science II — **Spring 2009, day section**
Prof. Marvin K. Nakayama

Print family (or last) name: ________________________________

Print given (or first) name: ________________________________

I have read and understand all of the instructions below, and I will obey the Academic Honor Code.

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Signature and Date

- This exam has 7 pages in total, numbered 1 to 7. Make sure your exam has all the pages.
- Note the number written on the upper right-hand corner of the first page. On the sign-up sheet being passed around, sign your name next to this number.
- This exam will be 1 hour and 25 minutes in length.
- This is a closed-book, closed-note exam.
- For all problems, follow these instructions:
  1. Give only your answers in the spaces provided. I will only grade what you put in the answer space, and I will take off points for any scratch work in the answer space. Use the scratch-work area or the backs of the sheets to work out your answers before filling in the answer space.
  2. For any proofs, be sure to provide a step-by-step argument, with justifications for every step.

Problem	1	2	3	4	5	Total
Points						

1. **[20 points]** For each of the following, circle TRUE if the statement is correct. Otherwise, circle FALSE

(a) TRUE FALSE — The language $\{ 0^n1^n \mid 0 \leq n \leq 1000 \}$ is regular.

(b) TRUE FALSE — If a language is context-free, then it must be regular.

(c) TRUE FALSE — If a language is regular, then it must be finite.

(d) TRUE FALSE — Nonregular languages are recognized by NFAs.

(e) TRUE FALSE — The class of context-free languages is closed under intersection.

(f) TRUE FALSE — If a language A has a regular expression, then $\overline{A}$ must be a context-free language.

(g) TRUE FALSE — A language has a regular expression if and only if it has an NFA.

(h) TRUE FALSE — The regular expression $(01^*0 \cup 1)^*0$ generates the language consisting of all strings over $\Sigma = \{0, 1\}$ having an odd number of 0's.

(i) TRUE FALSE — If a language A has a PDA, then A is generated by a context-free grammar in Chomsky normal form.

(j) TRUE FALSE — If A is a context-free language and B is a language such that $B \subseteq A$, then B must be a context-free language.

2. **[20 points]** Give short answers to each of the following parts. Be sure to define any notation that you use.

(a) Give an NFA with exactly three states for the language having regular expression 1*0*1*1. You only need to draw the picture.

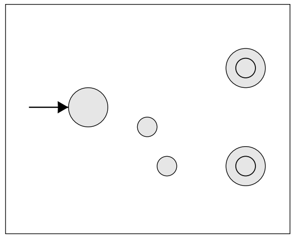
(b) Consider the following CFG $G = (V, \Sigma, R, S)$, with $V = \{S, X, Y\}$, $\Sigma = \{a, b\}$, start variable S, and rules R as follows:

$$
\begin{aligned}
S &\rightarrow XY \mid aX \mid \varepsilon \\
X &\rightarrow YXY \mid \varepsilon \\
Y &\rightarrow a \mid XS
\end{aligned}
$$

Note that G is not in Chomsky normal form. List all of the rules in G that violate Chomsky normal form. Explain your answer.

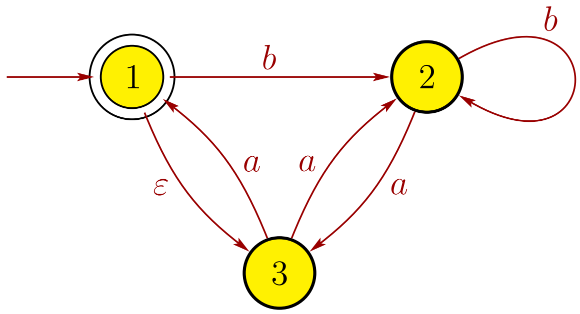
(c) Suppose that language A_1 is recognized by NFA N_1 below. Note that the transitions are not drawn in N_1. Draw a picture of an NFA for A_1^*.

N_1



(d) Suppose that language A_1 has CFG $G_1 = (V_1, \Sigma, R_1, S_1)$ and language A_2 has CFG $G_2 = (V_2, \Sigma, R_2, S_2)$. Give a CFG G_3 for $A_1 \circ A_2$ in terms of G_1 and G_2. You do not have to prove the correctness of your CFG G_2, but do not give just an example.

3. **[20 points]** Let N be the following NFA with $\Sigma = \{a, b\}$, and let $C = L(N)$.



(a) List the strings in C in lexicographic order. If C has more than 5 strings, list only the first 5 strings in C, followed by 3 dots.

(b) Give a DFA for C.

Scratch-work area

4. **[25 points]** Consider the alphabet $\Sigma = \{a, b\}$ and the language

$$L = \{ \ b^{3n} a^{2n} \mid n \geq 0 \ \}.$$

(a) Give a context-free grammar G for L. Be sure to specify G as a 4-tuple $G = (V, \Sigma, R, S)$.

(b) Give a PDA for L. You only need to draw the graph.

Scratch-work area

5. **[15 points]** Recall the pumping lemma for regular languages:

Theorem: If L is a regular language, then there exists a pumping length p where, if $s \in L$ with $|s| \geq p$, then there exists strings x, y, z such that $s = xyz$ and (i) $xy^iz \in L$ for each $i \geq 0$, (ii) $|y| \geq 1$, and (iii) $|xy| \leq p$.

Let $A = \{ a^{2n}c^{3n}b^{2n} \mid n \geq 0 \}$. Is A a regular or nonregular language? If A is regular, give a regular expression for A. If A is not regular, prove that it is a nonregular language.

Circle one: **Regular Language** **Nonregular Language**