CS 341, Spring 2009
Solutions for Midterm 2

1. (a) False, e.g., $\overline{A_{\text{TM}}}$ is not Turing-recognizable.
   (b) False, e.g., if $A = \{00, 11, 111\}$ and $B = \{00, 11\}$, then $\overline{A} \cap B = \emptyset$ but $A \neq B$. For A and B to be equal, we instead need $(\overline{A} \cap B) \cup (A \cap \overline{B}) = \emptyset$.
   (c) False. A TM M may loop on input w.
   (d) True, by Theorem 4.9.
   (e) False, by Theorem 4.8.
   (f) True, by Theorem 5.2.
   (g) False, by Theorem 4.11.
   (h) True, by Theorem 4.5.
   (i) False, by slide 4-39.
   (j) False, by Corollary 4.23.
2. (a) No, because $f(x) = f(z) = 2$.
   (b) Yes, because $f(y) = 1$ and $f(x) = 2$, so all members of B are hit by f.
   (c) No, because f is not one-to-one.
   (d) An algorithm is a Turing machine that always halts.
   (e) A language L_1 that is Turing-recognizable has a Turing machine M_1 that may loop forever on a string $w \notin L_1$. A language L_2 that is Turing-decidable has a Turing machine M_2 that always halts.
3. (a) $q_1010\#1 \quad xq_210\#1 \quad x1q_20\#1 \quad x10q_2\#1 \quad x10\#q_41 \quad x10\#1q_{\text{reject}}$
   (b) $q_11\#1 \quad xq_3\#1 \quad x\#q_51 \quad xq_6\#x \quad q_7x\#x \quad xq_1\#x \quad x\#q_8x \quad x\#xq_8 \quad x\#x\sqcup q_{\text{accept}}$
4. Homework 9, problem 1.
5. Define the language as

   $$C = \{\langle N, R\rangle \mid N \text{ is an NFA and } R \text{ is a regular expression with } L(N) = L(R)\}.$$

   Recall that the proof of Theorem 4.5 defines a Turing machine F that decides the language $EQ_{\text{DFA}} = \{\langle A, B\rangle \mid A \text{ and } B \text{ are DFAs and } L(A) = L(B)\}$. Then the following Turing machine T decides C:

   T = "On input $\langle N, R\rangle$, where N is an NFA and R is a regular expression:
   1. Convert N and R into equivalent DFAs D_1 and D_2 using the algorithms in the proof of Kleene's Theorem.
   2. Run TM F from Theorem 4.5 on input $\langle D_1, D_2\rangle$.
   3. If F accepts, *accept*. If F rejects, *reject*."
6. This is Theorem 5.1, whose proof is given on slide 5-8.