Midterm Exam 1
CS 341-007: Foundations of Computer Science II — **Fall 2022, face-to-face section**
Prof. Marvin K. Nakayama

Print family (or last) name: ____

Print given (or first) name: ____

I have read and understand all of the instructions below, and I will obey the University Policy on Academic Integrity.

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Signature and Date

- This exam has 8 pages in total, numbered 1 to 8. Make sure your exam has all the pages.
- Note the number written on the upper right-hand corner of the first page. On the sign-up sheet being passed around, print your name next to this number.
- This exam will be 1 hour and 20 minutes in length.
- This is a closed-book, closed-note exam. Electronic devices (e.g., cellphone, smart watch, calculator) are not allowed.
- For all problems, follow these instructions:
  1. Give only your answers in the spaces provided. Only what is written in the answer space will be graded, and points will be deducted for any scratch work in the answer space. Use the scratch-work area or the backs of the exam sheets to work out your answers before filling in the answer space.
  2. DFA stands for deterministic finite automaton; NFA stands for nondeterministic finite automaton; PDA stands for push-down automaton; CFG stands for context-free grammar.
  3. For any state diagrams that you draw, you must include **all states and transitions**.
  4. For any proofs, be sure to provide a step-by-step argument, with justifications for every step. If you are asked to prove a result X, you may use in your proof of X any other result Y without proving Y. However, make it clear what the other result Y is that you are using; e.g., write something like, "By the result that $A^{**} = A^*$, we know that ...."

Problem	1	2	3	4	5	Total
Points						

1. **[20 points, Multiple Choice]** For each of the following questions, circle the letter of the correct answer.

   1.1. The class of context-free languages satisfies which of the following:
   (a) it is closed under intersection.
   (b) it is closed under complementation.
   (c) it is closed under concatenation.
   (d) it contains every possible language.
   (e) it does not contain every regular language.
   (f) none of the above.

   1.2. Suppose that L_1 and L_2 are infinite regular languages. Then
   (a) $L_1 \cup L_2$ must be context-free.
   (b) $L_1 \cup L_2$ must be non-context-free.
   (c) $L_1 \cup L_2$ can be context-free and also it can be non-context-free.
   (d) $L_1 \cup L_2$ must be nonregular.
   (e) none of the above.

   1.3. If L_1 and L_2 are infinite non-regular languages, then
   (a) $L_1 \cap L_2$ must be non-regular.
   (b) $L_1 \cap L_2$ must be regular.
   (c) $L_1 \cap L_2$ can be regular and also it can be non-regular.
   (d) $L_1 \cap L_2$ must be context-free.
   (e) none of the above.

   1.4. If L is an infinite language, then
   (a) L must be regular.
   (b) L must be context-free, but not regular.
   (c) L must be non-context-free and nonregular.
   (d) L must be closed under Kleene star.
   (e) none of the above.

   1.5. The language $A = \{ b^n a^n \mid n \geq 0 \}$ satisfies which of the following?
   (a) A has regular expression $b^* a^*$.
   (b) A has regular expression $(ba)^*$.
   (c) A has CFG $G = (V, \Sigma, R, S)$, with $V = \{S\}$, $\Sigma = \{a, b\}$, $R = \{S \rightarrow bSa\}$, and starting variable S.
   (d) A is not context-free.
   (e) none of the above.

1.6. If A is a non-context-free language, then

(a) A must be finite.

(b) A must be regular.

(c) A must be closed under reversals.

(d) none of the above.

1.7. If a language L is recognized by a PDA, then

(a) L must be finite.

(b) L must be infinite.

(c) L can be finite and also it can be infinite.

(d) does not have a context-free grammar in Chomsky normal form.

(e) none of the above.

1.8. If a language L has a regular expression, then

(a) L must be a nonregular language.

(b) L must be a non-context-free language.

(c) L must have a context-free grammar.

(d) L must be closed under concatenation.

(e) none of the above.

1.9. If a finite number of strings is added to a nonregular language A, then the resulting language B satisfies which of the following?

(a) B must be a regular language.

(b) B must be a nonregular language.

(c) B must be a non-context-free language.

(d) B must have a context-free grammar.

(e) none of the above.

1.10. Let $\Sigma = \{0, 1\}$, and let L be the language of all non-empty strings over Σ that begin and end with the same symbol. Consider the following regular expressions:

(i) 01^*0

(ii) $0(0 \cup 1)^*0 \cup 1(0 \cup 1)^*1$

(iii) $(0 \cup 1)(0 \cup 1)^*(0 \cup 1)$

Which of the following statements is correct?

(a) Only regular expression (i) generates L.

(b) Only regular expression (ii) generates L.

(c) Only regular expression (iii) generates L.

(d) Only regular expressions (i) and (ii) generate L.

(e) Only regular expressions (i) and (iii) generate L.

(f) Only regular expressions (ii) and (iii) generate L.

(g) All 3 regular expressions generate L.

(h) None of the 3 regular expressions generates L.

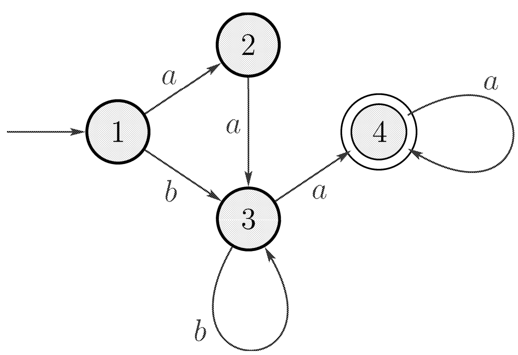
2. **[20 points]** Give short answers to each of the following parts. Each answer should be at most a few sentences. Be sure to define any notation that you use.

(a) Let $\Sigma = \{a, b\}$, and let A be the set of strings $w \in \Sigma^*$ such that $|w| \geq 4$ and the second-to-last symbol is a. If $w = w_1 w_2 \cdots w_n$ with $n \geq 2$ and each $w_i \in \Sigma$, then $|w| = n$ and w_{n-1} is the second-to-last symbol of w. Give a regular expression for A.

Answer: ____

(b) Give a regular expression for the language recognized by the NFA below.

Answer: ____



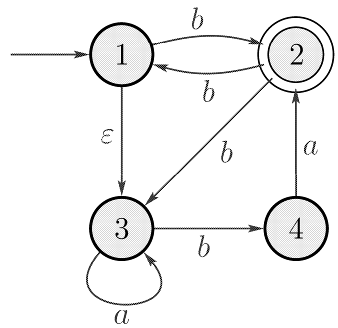
(c) Suppose that we are in the process of converting a CFG G with $\Sigma = \{0, 1\}$ into Chomsky normal form. We have already applied some steps in the process, and we currently have the following CFG:

$$\begin{aligned} S_0 &\rightarrow S \\ S &\rightarrow 1SA0A \mid 0AS1S \mid \varepsilon \\ A &\rightarrow 10S1 \mid \varepsilon \end{aligned}$$

In the next step, we want to remove the ε-rule $A \rightarrow \varepsilon$. Give the CFG after carrying out just this one step.

(d) Suppose that A_1 is a language defined by a DFA $M_1 = (Q_1, \Sigma, \delta_1, q_1, F_1)$ and A_2 is a language defined by a DFA $M_2 = (Q_2, \Sigma, \delta_2, q_2, F_2)$, where the alphabet Σ is the same for both languages. Let $A_3 = A_1 \cap A_2$. Give a DFA M_3 for A_3 in terms of M_1 and M_2. You do not have to prove the correctness of your DFA M_3, but do not give just an example.

3. [**15 points**] Let N be the following NFA with $\Sigma = \{a, b\}$, and let $C = L(N)$.



Give a DFA for C. You only need to draw the state diagram (graph); do not give the 5-tuple.

Scratch-work area

4. [**30 points**] Consider the alphabet $\Sigma = \{a, b, c\}$ and the language

$$L = \{ \, b^i a^j c^k \mid i, j, k \geq 0 \text{ and } i = j + k \, \}.$$

(a) Give a context-free grammar G for L. Be sure to specify G as a 4-tuple $G = (V, \Sigma, R, S)$.

(b) Give a PDA for L. You only need to draw the state diagram (graph); you do not need to give the 6-tuple for your PDA.

Scratch-work area

5. [**15 points**] Recall the pumping lemma for regular languages:

Theorem: If L is a regular language, then there exists a pumping length p where, if $s \in L$ with $|s| \geq p$, then s can be split into three pieces $s = xyz$ such that (i) $xy^iz \in L$ for each $i \geq 0$, (ii) $|y| \geq 1$, and (iii) $|xy| \leq p$.

Let $A = \{ b^i a^j c^k \mid i, j, k \geq 0 \text{ and } i = j + k \}$. Is A a regular or nonregular language? If A is regular, give a regular expression for A. If A is not regular, prove that it is a nonregular language.

Circle one: **Regular Language** **Nonregular Language**