Midterm Exam 1
CS 341-**010**: Foundations of Computer Science II — **Spring 2024, face-to-face section**
Prof. Marvin K. Nakayama

Print family (or last) name: ______________________________

Print given (or first) name: ______________________________

I have read and understand all of the instructions below, and I will obey the University Policy on Academic Integrity.

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Signature and Date

- This exam has 8 pages in total, numbered 1 to 8. Make sure your exam has all the pages.
- Note the number written on the upper right-hand corner of the first page. On the sign-up sheet being passed around, print your name next to this number.
- This exam will be 1 hour and 20 minutes in length.
- This is a closed-book, closed-note exam. Unauthorized materials, including notes and electronic devices (e.g., cellphone, smart watch, calculator), are not allowed.
- For all problems, follow these instructions:
  1. Give only your answers in the spaces provided. **Be sure to clearly indicate your answers.** Only what is written in the answer space will be graded, and points will be deducted for any scratch work in the answer space. Use the backs of the exam sheets to work out your answers before filling in an answer space.
  2. DFA stands for deterministic finite automaton; NFA stands for nondeterministic finite automaton; PDA stands for push-down automaton; CFG stands for context-free grammar.
  3. For any state diagrams that you draw, you must **include all states and transitions**.
  4. For any proofs, be sure to provide a step-by-step argument, with justifications for every step. If you are asked to prove a result X, you may use in your proof of X any other result Y covered in lecture or the HW without proving Y. However, make it clear what the other result Y is that you are using; e.g., write something like, "By the result that $A^{**} = A^*$, we know that ...."

Problem	1	2	3	4	Total
Points					

1. [**40 points, Multiple Choice**] For each of the following questions, circle the letter of the correct answer.

1.1. If L_1 and L_2 are non-context-free languages, then
(a) $L_1 \cap L_2$ must be non-context-free.
(b) $L_1 \cap L_2$ must be context-free.
(c) $L_1 \cap L_2$ can be context-free and also $L_1 \cap L_2$ can be non-context-free.
(d) $L_1 \cap L_2$ must be nonregular.
(e) none of the above.

1.2. If A is any language, then
(a) A^* must be regular.
(b) $\overline{A}$ must be regular.
(c) $\overline{A} \cup A$ must be regular.
(d) all of the above are true.
(e) none of the above are true.

1.3. The language $A = \{\, 0^{3n}1^{2n}0^{n} \mid n \geq 0 \,\}$ satisfies which of the following?
(a) A has regular expression $(000)^*(11)^*(0)^*$.
(b) A is recognized by some NFA.
(c) A is recognized by some PDA.
(d) A is not context-free.
(e) none of the above.

1.4. If language A is recognized by an NFA and language B has a regular expression, then
(a) $A \circ B$ must be a nonregular language.
(b) $A \cup B$ is not recognized by any DFA.
(c) $A \cap B$ must be a non-context-free language.
(d) $A \cap B$ must be recognized by a PDA.
(e) none of the above.

1.5. If an infinite number of strings is removed from a context-free language A, then the resulting language B satisfies which of the following?
(a) B must be a regular language.
(b) B must be a nonregular language.
(c) B must be a non-context-free language.
(d) none of the above.

1.6. If A is a regular language, then
(a) A is closed under concatenations.
(b) $A \circ A$ must be context-free.
(c) A must be finite.
(d) all of the above.
(e) none of the above.

1.7. The class of languages recognized by NFAs satisfies which of the following?
(a) it is closed under complementation.
(b) it is not closed under intersection.
(c) it contains every possible language.
(d) it includes every context-free language.
(e) none of the above.

1.8. The language $A = \{ 1^i 0^j \mid i \geq 0, j \geq 0 \}$ satisfies which of the following?
(a) A is finite.
(b) A is regular.
(c) A is nonregular but context-free.
(d) A is non-context-free.
(e) none of the above.

1.9. Suppose that A, B, and C are context-free languages. Then
(a) $\overline{A}(B^* \cup C)$ must be context-free.
(b) $\overline{A}(B \cup C^*)$ must be non-context-free.
(c) $\overline{A}(B \cup C) = (C \cup B)\overline{A}$.
(d) $A^*(B \cup C)$ must be context-free.
(e) none of the above.

1.10. Let $\Sigma = \{0, 1\}$, and let A be the language of all strings over Σ that have even length or an odd number of 1's. Consider the following regular expressions:
(i) $R_1 = ((0 \cup 1)(0 \cup 1))^* \cup (0^*10^* \cup 0^*1)(0^*10^*1)^*$
(ii) $R_2 = (00 \cup 01 \cup 10 \cup 11)^* \cup 0^*1(0 \cup 10^*1)^*$
(iii) $R_3 = 0^*10^*(0^*10^*10^*)^* \cup ((0 \cup 1)(0 \cup 1))^*$
Which of the following statements is correct?
(a) $A = L(R_1)$, $A \neq L(R_2)$, and $A \neq L(R_3)$.
(b) $A \neq L(R_1)$, $A = L(R_2)$, and $A \neq L(R_3)$.
(c) $A \neq L(R_1)$, $A \neq L(R_2)$, and $A = L(R_3)$.
(d) $A = L(R_1)$, $A = L(R_2)$, and $A \neq L(R_3)$.
(e) $A = L(R_1)$, $A \neq L(R_2)$, and $A = L(R_3)$.
(f) $A \neq L(R_1)$, $A = L(R_2)$, and $A = L(R_3)$.
(g) $A = L(R_1)$, $A = L(R_2)$, and $A = L(R_3)$.
(h) $A \neq L(R_1)$, $A \neq L(R_2)$, and $A \neq L(R_3)$.

1.11. The language $A = \{ w \in \{ 0, 1 \}^* \mid w = w^{\mathcal{R}}, |w| \text{ is even} \}$ satisfies which of the following?
(a) A has regular expression $(0^*1^*)^*(1^*0^*)^*$.
(b) A has context-free grammar $G = (V, \Sigma, R, S)$, with $\Sigma = \{ 0, 1 \}$, $V = \{ S \}$, starting variable S, and rules $R = \{ S \to 0S0 \mid 1S1 \mid 0 \mid 1 \mid \varepsilon \}$.
(c) A has context-free grammar $G = (V, \Sigma, R, S)$, with $\Sigma = \{ 0, 1 \}$, $V = \{ S \}$, starting variable S, and rules $R = \{ S \to 0S0 \mid 1S1 \mid \varepsilon \}$.
(d) all of the above.
(e) none of the above.

1.12. The language $A = \{ b^i a^j \mid i \geq 0, j \geq 0, i = j \}$ satisfies which of the following?

(a) A has regular expression b^*a^*.

(b) A has regular expression $(ba)^*$.

(c) A has CFG $G = (V, \Sigma, R, S)$, with $V = \{S\}$, $\Sigma = \{a, b\}$, $R = \{ S \to bSa \}$, and starting variable S.

(d) A is not context-free.

(e) none of the above.

1.13. If a language L is recognized by a PDA, then

(a) must have a regular expression.

(b) L must be finite.

(c) L must be infinite.

(d) L can be finite and also L can be infinite.

(e) none of the above.

1.14. If a language L has a regular expression that includes a Kleene-star $*$, then

(a) $\varepsilon \in L$.

(b) L must have a context-free grammar in Chomsky normal form.

(c) L must be an infinite language.

(d) all of the above.

(e) none of the above.

1.15. For the alphabet $\Sigma = \{ [,] \}$, consider the language A of strings in Σ^* of properly balanced left and right brackets: every left bracket can be paired with a unique subsequent right bracket, and every right bracket can be paired with a unique preceding left bracket. Moreover, the string between any such pair has the same property. For example, $[\,]\,[\,[\,[\,]\,[\,]\,]\,[\,]\,] \in A$. Which of the following statements is correct?

(a) A is finite.

(b) A is regular.

(c) A is nonregular but context-free.

(d) A is non-context-free.

(e) none of the above.

1.16. If A and B are languages with A nonregular and $A \subseteq B$, then

(a) B must be regular.

(b) B must be nonregular.

(c) B must be a context-free language.

(d) B must be non-context-free.

(e) none of the above.

1.17. If A is a language and there is some string $w \in A$ with $w \neq \varepsilon$, then

(a) A^* must be infinite.

(b) $A \circ A = A$.

(c) $A^+ = A^*$.

(d) $A^* = A$.

(e) none of the above.

1.18. Which of the following statements is true?

(a) $\emptyset = \{\varepsilon\}$

(b) $\emptyset = \varepsilon$

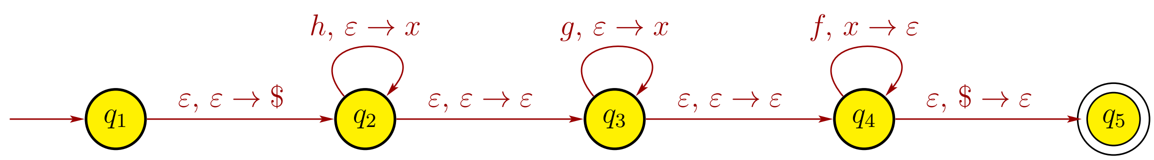
(c) $\emptyset^* = \{\varepsilon\}$

(d) $\emptyset^* = \emptyset$

(e) all of the above.

(f) none of the above.

1.19. For $\Sigma = \{f, g, h\}$, let $A \subseteq \Sigma^*$ be the language recognized by the following PDA:



Consider the following languages:

(i) $L_1 = \{\, h^i g^j f^k \mid i, j, k \geq 0 \text{ and } i = j + k \,\}$

(ii) $L_2 = \{\, h^i g^j f^k \mid i, j, k \geq 0 \text{ and } i + k = j \,\}$

(iii) $L_3 = \{\, h^i g^j f^k \mid i, j, k \geq 0 \text{ and } i = j = k \,\}$

Which of the following statements is correct?

(a) $A = L_1$, $A \neq L_2$, and $A \neq L_3$.

(b) $A \neq L_1$, $A = L_2$, and $A \neq L_3$.

(c) $A \neq L_1$, $A \neq L_2$, and $A = L_3$.

(d) $A = L_1$, $A = L_2$, and $A \neq L_3$.

(e) $A = L_1$, $A \neq L_2$, and $A = L_3$.

(f) $A \neq L_1$, $A = L_2$, and $A = L_3$.

(g) $A = L_1$, $A = L_2$, and $A = L_3$.

(h) $A \neq L_1$, $A \neq L_2$, and $A \neq L_3$.

1.20. If A is a non-context-free language and B is a context-free language, then

(a) A must be nonregular.

(b) B must be nonregular.

(c) $\overline{B}$ must be infinite.

(d) $A \cup B$ must be non-context-free.

(e) all of the above.

(f) none of the above.

2. **[15 points]** We say that a DFA M for a language A is *minimal* if there does not exist another DFA M' for A such that M' has strictly fewer states than M. Suppose that $M = (Q, \Sigma, \delta, q_0, F)$ is a minimal DFA for A. Using M, we construct a DFA $\overline{M}$ for the complement $\overline{A}$ as $\overline{M} = (Q, \Sigma, \delta, q_0, Q - F)$. Prove that $\overline{M}$ is a minimal DFA for $\overline{A}$.

3. **[25 points]** Recall the pumping lemma for context-free languages:

Theorem: If L is a context-free language, then there exists a pumping length p where, if $s \in L$ with $|s| \geq p$, then s can be split into five pieces $s = uvxyz$ such that (i) $uv^ixy^iz \in L$ for each $i \geq 0$, (ii) $|vy| \geq 1$, and (iii) $|vxy| \leq p$.

For $\Sigma = \{a, b, c\}$, let

$$A = \{\, bc^nbabc^nb \mid n \geq 0 \,\}.$$

Is A a context-free or non-context-free language? If A is context-free, give a CFG (4-tuple $G = (V, \Sigma, R, S)$) **and** PDA (only state diagram) for A. If A is not context-free, prove that it is a non-context-free language.

Circle one: **Context-Free Language** **Non-Context-Free Language**

4. [**20 points**] Recall the pumping lemma for regular languages:

Theorem: If L is a regular language, then there exists a pumping length p where, if $s \in L$ with $|s| \geq p$, then s can be split into three pieces $s = xyz$ such that (i) $xy^iz \in L$ for each $i \geq 0$, (ii) $|y| \geq 1$, and (iii) $|xy| \leq p$.

Let $A = \{ w \in \{0,1\}^* \mid n_{01}(w) = n_{10}(w) \}$, where $n_s(w)$ is the number of occurrences of the substring $s \in \{0,1\}^*$ in w. For example, the string $w_1 = 0001101100$ has $n_{01}(w_1) = 2$ and $n_{10}(w_1) = 2$, so $w_1 \in A$. Also, the string $w_2 = 00011011001$ has $n_{01}(w_2) = 3$ and $n_{10}(w_2) = 2$, so $w_2 \notin A$.

Is A a regular or nonregular language? If A is regular, give a regular expression **and** DFA (only state diagram) for A. If A is not regular, prove that it is a nonregular language.

Circle one: **Regular Language** **Nonregular Language**