Midterm Exam 1
CS 341: Foundations of Computer Science II — **Fall 2025, Sections 005 and HM1**
Prof. Marvin K. Nakayama

Print family (or last) name: ______________________________

Print given (or first) name: ______________________________

I have read and understand all of the instructions below, and I will obey the University Policy on Academic Integrity.

______________________________ **Section Number: 005 HM1**
Signature and Date **(circle one)**

- This exam has 8 pages in total, numbered 1 to 8. Make sure your exam has all the pages.
- Note the number written on the upper right-hand corner of the first page. On the sign-up sheet being passed around, print your name next to this number.
- This exam will be 1 hour and 20 minutes in length.
- This is a closed-book, closed-note exam. Unauthorized materials, including notes and electronic devices (e.g., cellphone, smart watch, smart glasses, calculator, headphones), are not allowed.
- For all problems, follow these instructions:
  1. Give only your answers in the spaces provided. **Be sure to clearly indicate your answers.** Only what is written in the answer space will be graded, and points will be deducted for any scratch work in the answer space. Use the backs of the exam sheets to work out your answers before filling in an answer space.
  2. DFA stands for deterministic finite automaton; NFA stands for nondeterministic finite automaton; PDA stands for push-down automaton; CFG stands for context-free grammar.
  3. For any state diagrams that you draw, you must **include all states and transitions**.
  4. For any proofs, be sure to provide a step-by-step argument, with justifications for every step. If you are asked to prove a result X, you may use in your proof of X any other result Y covered in lecture or the HW without proving Y. However, make it clear what the other result Y is that you are using; e.g., write something like, "By the result that $A^{**} = A^*$, we know that ...."

Problem	1	2	3	4	5	Total
Points						

1. **[30 points, Multiple Choice]** For each of the following questions, circle the letter of the correct answer.

   1.1. If A and B are languages with B regular and $A \subseteq B$, then
   (a) A must be regular.
   (b) A must be nonregular.
   (c) A must be a context-free language.
   (d) A must be non-context-free.
   (e) none of the above.

   1.2. The class of context-free languages satisfies which of the following:
   (a) it is closed under intersection.
   (b) it is closed under complementation.
   (c) it is closed under concatenation.
   (d) it contains every possible language.
   (e) it does not contain every regular language.
   (f) none of the above.

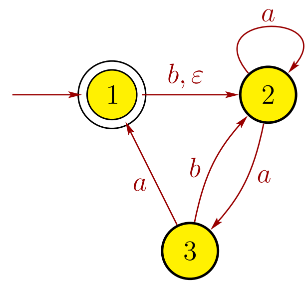
   1.3. Suppose that L_1 and L_2 are infinite regular languages. Then
   (a) $L_1 \cup L_2$ must be context-free.
   (b) $L_1 \cup L_2$ must be non-context-free.
   (c) $L_1 \cup L_2$ can be context-free and also it can be non-context-free.
   (d) $L_1 \cup L_2$ must be nonregular.
   (e) none of the above.

   1.4. If L is an infinite language over an alphabet Σ, then
   (a) L must be regular.
   (b) L must be context-free, but not regular.
   (c) L must be non-context-free and nonregular.
   (d) L must contain every string in Σ^*.
   (e) none of the above.

   1.5. The language $A = \{ b^i a^j \mid i \geq 0, j \geq 0, \text{ and } j = 2i \}$ satisfies which of the following?
   (a) A has regular expression $b^*(aa)^*$.
   (b) A has regular expression $(baa)^*$.
   (c) A has CFG $G = (V, \Sigma, R, S)$, with $V = \{S\}$, $\Sigma = \{a, b\}$, $R = \{S \rightarrow bSaa\}$, and starting variable S.
   (d) A is not context-free.
   (e) none of the above.

   1.6. If a language A has a regular expression, then
   (a) A must be finite.
   (b) A must be infinite.
   (c) A must have a DFA.
   (d) none of the above.

1.7. Which choice below is correct for the following NFA N?



(a) $ab \in L(N)$.
(b) $\varepsilon \in L(N)$.
(c) $\varepsilon \notin L(N)$.
(d) none of the above.

1.8. If a finite number of strings is added to a nonregular language A, then the resulting language B satisfies which of the following?
(a) B must be a regular language.
(b) B must be a nonregular language.
(c) B must be a non-context-free language.
(d) B must have a context-free grammar.
(e) none of the above.

1.9. Let $\Sigma = \{0, 1\}$, and let L be the language of all non-empty strings over Σ that begin and end with the same symbol. Consider the following regular expressions:
R1: 01^*0
R2: $0(0 \cup 1)^*0 \cup 1(0 \cup 1)^*1 \cup (0 \cup 1)$
R3: $(0 \cup 1)(0 \cup 1)^*(0 \cup 1)$
Which of the following statements is correct?
(a) R1 generates L, but R2 and R3 do not.
(b) R2 generates L, but R1 and R3 do not.
(c) R3 generates L, but R1 and R2 do not.
(d) R1 and R2 generate L, but R3 does not.
(e) R1 and R3 generate L, but R2 does not.
(f) R2 and R3 generate L, but R1 does not.
(g) All 3 regular expressions generate L.
(h) None of the 3 regular expressions generates L.

1.10. The language $A = \{ w \in \{ 0, 1 \}^* \mid w = w^{\mathcal{R}},\ |w| \text{ is even} \}$ satisfies which of the following?
(a) A has regular expression $(0^*1^*)^*(1^*0^*)^*$.
(b) A has context-free grammar $G = (V, \Sigma, R, S)$, with $\Sigma = \{ 0, 1 \}$, $V = \{ S \}$, starting variable S, and rules $R = \{ S \to 0S0 \mid 1S1 \mid 0 \mid 1 \mid \varepsilon \}$.
(c) A has context-free grammar $G = (V, \Sigma, R, S)$, with $\Sigma = \{ 0, 1 \}$, $V = \{ S \}$, starting variable S, and rules $R = \{ S \to 0S0 \mid 1S1 \mid \varepsilon \}$.
(d) all of the above.
(e) none of the above.

1.11. If A and B are finite languages defined over the same alphabet Σ, then
(a) $\overline{A} \circ B$ must be finite.
(b) $\overline{A} \circ B$ must be infinite.
(c) $A \cap \overline{B}$ must be context-free.
(d) $\overline{A} \cup \overline{B}$ must be nonregular.
(e) none of the above.

1.12. The language $A = \{ a^n b^n \mid 0 \leq n \leq 100 \}$ satisfies which of the following?

(a) A has a regular expression.
(b) A has a PDA but not a regular expression.
(c) A does not have a PDA.
(d) A does not have an NFA.
(e) none of the above.

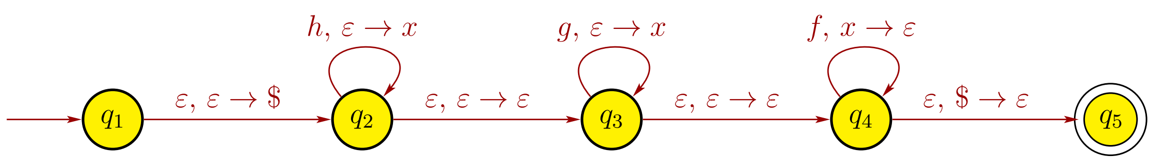
1.13. If A is a non-context-free language and B is a context-free language, then

(a) A must be nonregular.
(b) B must be nonregular.
(c) $\overline{B}$ must be infinite.
(d) $A \cup B$ must be non-context-free.
(e) all of the above.
(f) none of the above.

1.14. Which of the following statements is true?

(a) $\emptyset = \{\varepsilon\}$
(b) $\emptyset = \varepsilon$
(c) $\emptyset^* = \{\varepsilon\}$
(d) $\emptyset^* = \emptyset$
(e) all of the above.
(f) none of the above.

1.15. For $\Sigma = \{f, g, h\}$, let $A \subseteq \Sigma^*$ be the language recognized by the following PDA:



Consider the following languages:

(i) $L_1 = \{ h^i g^j f^k \mid i, j, k \geq 0 \text{ and } i = j + k \}$
(ii) $L_2 = \{ h^i g^j f^k \mid i, j, k \geq 0 \text{ and } i + k = j \}$
(iii) $L_3 = \{ h^i g^j f^k \mid i, j, k \geq 0 \text{ and } i = j = k \}$

Which of the following statements is correct?

(a) $A = L_1$, $A \neq L_2$, and $A \neq L_3$.
(b) $A \neq L_1$, $A = L_2$, and $A \neq L_3$.
(c) $A \neq L_1$, $A \neq L_2$, and $A = L_3$.
(d) $A = L_1$, $A = L_2$, and $A \neq L_3$.
(e) $A = L_1$, $A \neq L_2$, and $A = L_3$.
(f) $A \neq L_1$, $A = L_2$, and $A = L_3$.
(g) $A = L_1$, $A = L_2$, and $A = L_3$.
(h) $A \neq L_1$, $A \neq L_2$, and $A \neq L_3$.

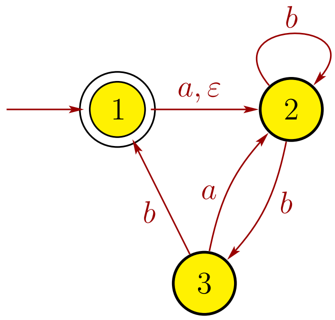
2. **[15 points]** Give short answers to each of the following parts. Each answer should be at most a few sentences. Be sure to define any notation that you use.

   (a) Let $\Sigma = \{c, d\}$ (**note the alphabet!**), and let A be the set of strings $w \in \Sigma^*$ such that $|w|$ is even and w ends in c, where $|w|$ denotes the length of w. Give a regular expression for A.

   Answer: ____

   (b) Suppose that language A_1 has CFG $G_1 = (V_1, \Sigma, R_1, S_1)$ and language A_2 has CFG $G_2 = (V_2, \Sigma, R_2, S_2)$. Give a CFG G_3 for A_2^* in terms of G_1 and G_2. You do not have to prove the correctness of your CFG for A^*, but do not give just an example.

3. **[15 points]** Let N be the following NFA with $\Sigma = \{a, b\}$, and let $C = L(N)$.



Give a DFA for C. You only need to draw the state diagram (graph); do not give the 5-tuple.

4. **[25 points]** For $\Sigma = \{a, b, c\}$, let

$$A = \{\, b^{2n} c^{3k} a^n \mid n \geq 0, k \geq 0 \,\}.$$

(a) Give a context-free grammar G for L. Be sure to specify G as a 4-tuple $G = (V, \Sigma, R, S)$.

(b) Give a PDA for L. You only need to draw the state diagram (graph).

Scratch-work area

5. **[15 points]** Recall the pumping lemma for regular languages:

Theorem: If L is a regular language, then there exists a pumping length p where, if $s \in L$ with $|s| \geq p$, then s can be split into three pieces $s = xyz$ such that (i) $xy^iz \in L$ for each $i \geq 0$, (ii) $|y| \geq 1$, and (iii) $|xy| \leq p$.

For $\Sigma = \{e, f\}$ (**note the alphabet!**), let $A = \{\, www \mid w \in \Sigma^* \,\}$.

Is A a regular or nonregular language? If A is regular, give a regular expression **and** DFA (only state diagram) for A. If A is not regular, prove that it is a nonregular language.

Circle one: **Regular Language** **Nonregular Language**