

Exam 1

CS 341: Foundations of Computer Science II — **Fall 2026, Section H01**

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Print family (or last) name: _____

Print given (or first) name: _____

I have read and understand all of the instructions below, and I will obey the University Policy on Academic Integrity.

Signature and Date

- This exam has 9 pages in total, numbered 1 to 9. Make sure your exam has all the pages.
- Note the number written on the upper right-hand corner of the first page. On the sign-up sheet being passed around, print your name next to this number.
- This exam will be 1 hour and 20 minutes in length.
- This is a closed-book, closed-note exam. Unauthorized materials, including notes and electronic devices (e.g., cellphone, smart watch, smart glasses, calculator, headphones), are not allowed.
- For all problems, follow these instructions:
 1. Give only your answers in the spaces provided. **Be sure to clearly indicate your answers.** Only what is written in the answer space will be graded, and points will be deducted for any scratch work in the answer space. Use the backs of the exam sheets to work out your answers before filling in an answer space.
 2. DFA stands for deterministic finite automaton; NFA stands for nondeterministic finite automaton; PDA stands for push-down automaton; CFG stands for context-free grammar.
 3. For any state diagrams that you draw, you must **include all states and transitions**.
 4. For any proofs, be sure to provide a step-by-step argument, with justifications for every step. If you are asked to prove a result X, you may use in your proof of X any other result Y covered in lecture or the HW without proving Y. However, make it clear what the other result Y is that you are using; e.g., write something like, “By the result that $A^{**} = A^*$, we know that ...”

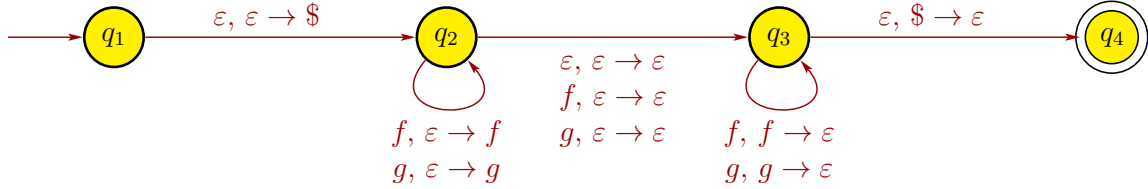
Problem	1	2	3	4	5	Total
Points						

1. **[30 points, Multiple Choice]** For each of the following questions, circle the letter of the correct answer.
- 1.1. If a language A has a regular expression and a language B is finite, then
- (a) A is closed under concatenation.
 - (b) $A^* \cap \overline{B}$ must be regular.
 - (c) A must be finite.
 - (d) all of the above.
 - (e) none of the above.
- 1.2. If L_1 and L_2 are non-context-free languages, then
- (a) $L_1 \cap L_2$ must be non-context-free.
 - (b) $L_1 \cap L_2$ must be context-free.
 - (c) $L_1 \cap L_2$ can be context-free and also $L_1 \cap L_2$ can be non-context-free.
 - (d) $L_1 \cap L_2$ must be nonregular.
 - (e) none of the above.
- 1.3. For alphabet $\Sigma = \{0, 1\}$, the language $A = \{ww \mid w \in \Sigma^*\}$ satisfies which of the following?
- (a) A has regular expression $(0 \cup 1)^*(0 \cup 1)^*$.
 - (b) A is recognized by some PDA.
 - (c) A is closed under concatenation.
 - (d) A is closed under reversal.
 - (e) none of the above.
- 1.4. The class of languages recognized by NFAs satisfies which of the following?
- (a) it is not closed under intersection.
 - (b) it is closed under complementation.
 - (c) it includes every nonregular language.
 - (d) it includes every context-free language.
 - (e) none of the above.
- 1.5. The language $A = \{1^i 0^j \mid i \geq 0, j \geq 0\}$ satisfies which of the following?
- (a) A is finite.
 - (b) A is regular, but not finite.
 - (c) A is nonregular but context-free.
 - (d) A is non-context-free.
 - (e) none of the above.
- 1.6. Suppose that A , B , and C are regular languages. Then
- (a) $\overline{B}(A^* \cup C)$ must be context-free.
 - (b) $(B \cap C^*)\overline{A}$ must be context-free.
 - (c) $A^*(\overline{B} \cup C)$ must be context-free.
 - (d) $(B \cap \overline{C})A^*$ must be context-free.
 - (e) all of the above.
 - (f) none of the above.

1.7. If A and B are languages with $A \subseteq B$ and B context-free, then

- (a) A must be regular.
- (b) A must be nonregular.
- (c) A must be a context-free language.
- (d) A must be non-context-free.
- (e) none of the above.

1.8. For $\Sigma = \{f, g\}$, let $A \subseteq \Sigma^*$ be the language recognized by the following PDA:



Consider the following languages:

- (i) $L_1 = \{ w \in \Sigma^* \mid w = w^R \}$
- (ii) $L_2 = \{ w \in \Sigma^* \mid w = w^R \text{ and } |w| \text{ is odd} \}$
- (iii) $L_3 = \{ wxw \mid w \in \Sigma^*, x \in \Sigma \}$

Which of the following statements is correct?

- (a) $A \neq L_1$, $A \neq L_2$, and $A \neq L_3$.
- (b) $A = L_1$, $A \neq L_2$, and $A \neq L_3$.
- (c) $A \neq L_1$, $A = L_2$, and $A \neq L_3$.
- (d) $A \neq L_1$, $A \neq L_2$, and $A = L_3$.
- (e) $A = L_1$, $A = L_2$, and $A \neq L_3$.
- (f) $A = L_1$, $A \neq L_2$, and $A = L_3$.
- (g) $A \neq L_1$, $A = L_2$, and $A = L_3$.
- (h) $A = L_1$, $A = L_2$, and $A = L_3$.

1.9. The language $A = \{ b^n a^n b^k \mid n \geq 0, k \geq 0 \}$ satisfies which of the following?

- (a) A has regular expression $b^* a^* b^*$.
- (b) A has regular expression $(ba)^* b^*$.
- (c) A has CFG $G = (V, \Sigma, R, S)$, with $V = \{S\}$, $\Sigma = \{a, b\}$, $R = \{ S \rightarrow bSa \mid bS \mid \varepsilon \}$, and starting variable S .
- (d) A is not context-free.
- (e) none of the above.

1.10. Suppose that A , B , and C are context-free languages. Then

- (a) $\overline{A}(B^* \cup C)$ must be context-free.
- (b) $\overline{A}(B \cap C^*)$ must be non-context-free.
- (c) $\overline{A}(B \cup C) = (C \cup B)\overline{A}$.
- (d) $A^*(B \cup C)$ must be context-free.
- (e) all of the above.
- (f) none of the above.

1.11. Let $\Sigma = \{0, 1\}$, and let A be the language of all strings over Σ that have an odd number of 1's or even length. Consider the following regular expressions:

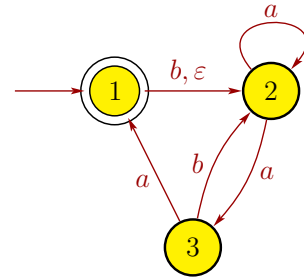
- $R_1 = (00 \cup 01 \cup 10 \cup 11)^* \cup 0^*1(0 \cup 10^*1)^*$
- $R_2 = ((0 \cup 1)(0 \cup 1))^* \cup (0^*10^* \cup 0^*1)(0^*10^*1)^*$
- $R_3 = (0^*10^*10^*)^*0^*10^* \cup ((0 \cup 1)(0 \cup 1))^*$

Which of the following statements is correct?

- (a) $A \neq L(R_1)$, $A \neq L(R_2)$, and $A \neq L(R_3)$.
- (b) $A = L(R_1)$, $A \neq L(R_2)$, and $A \neq L(R_3)$.
- (c) $A \neq L(R_1)$, $A = L(R_2)$, and $A \neq L(R_3)$.
- (d) $A \neq L(R_1)$, $A \neq L(R_2)$, and $A = L(R_3)$.
- (e) $A = L(R_1)$, $A = L(R_2)$, and $A \neq L(R_3)$.
- (f) $A = L(R_1)$, $A \neq L(R_2)$, and $A = L(R_3)$.
- (g) $A \neq L(R_1)$, $A = L(R_2)$, and $A = L(R_3)$.
- (h) $A = L(R_1)$, $A = L(R_2)$, and $A = L(R_3)$.

1.12. Which choice below is correct for the following NFA N ?

- (a) $aa \notin L(N)$.
- (b) $\varepsilon \in L(N)$.
- (c) $|L(N)| < \infty$.
- (d) all of the above.
- (e) none of the above.



1.13. If a finite number of strings is added to a nonregular language A , then the resulting language B satisfies which of the following?

- (a) B must be a regular language.
- (b) B must be a nonregular language.
- (c) B must be a non-context-free language.
- (d) B must have a context-free grammar.
- (e) none of the above.

1.14. The language $A = \{a^i b^j \mid 0 \leq i \leq j \leq 50\}$ satisfies which of the following?

- (a) A has a regular expression.
- (b) A has a PDA but not a regular expression.
- (c) A does not have a PDA.
- (d) A does not have an NFA.
- (e) none of the above.

1.15. Which of the following statements is true?

- (a) $\emptyset = \{\varepsilon\}$
- (b) $\emptyset^* = \{\varepsilon\}$
- (c) $\emptyset = \varepsilon$
- (d) $\emptyset^* = \emptyset$
- (e) all of the above.
- (f) none of the above.

2. **[25 points]** Give short answers to each of the following parts. Each answer should be at most a few sentences or a single drawing. Be sure to define any notation that you use.
- (a) **[10 points]** Suppose that language A_1 has CFG $G_1 = (V_1, \Sigma, R_1, S_1)$ and language A_2 has CFG $G_2 = (V_2, \Sigma, R_2, S_2)$. Give a CFG G_3 for $A_3 = A_2 \circ A_1 \cup A_1^*$ in terms of G_1 and G_2 . You do not have to prove the correctness of your CFG G_3 , but do not give just an example.

- (b) **[15 points]** Define a language L as follows. First define $\Omega = \{\mathbf{a}, \mathbf{b}, \mathbf{c}, \dots, \mathbf{z}\}$ as the set of lower-case Roman letters. Also, define $\Lambda = \{.\}$, and let $\Sigma = \Omega \cup \Lambda$; i.e., Σ is the set consisting of all the lower-case Roman letters and the dot (or period). Define the following sets of strings:

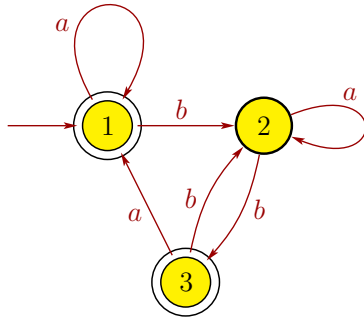
- $S_1 = \Omega\Omega^*$
- $S_2 = \Lambda\Omega\Omega^*$
- $S_3 = \{.co\}$
- $S_4 = \{.com\} \cup \{.cm\}$

Give a DFA for

$$L = S_1S_2^*(S_3 \cup S_4)$$

Give only the DFA drawing (state diagram); do not specify the 5-tuple. To simplify your DFA drawing, you should omit any edges going from any state q to a “trap state” (i.e., a non-accepting state from which the DFA can never leave). All other edges must be included in your drawing. Clearly identify which state is the trap state in the drawing of your DFA, and your drawing should include a note stating that all edges not specified go to a trap state. You will lose points if your DFA has more states than necessary or is overly complicated.

3. [10 points] Let M be the following DFA with $\Sigma = \{a, b\}$. Give a regular expression for $L(M)$.



4. [20 points] For $\Sigma = \{d, h, u\}$ (note the alphabet), let

$$L = \{ h^i u^j d^k \mid i, j, k \geq 0 \text{ and } i = j + k \}.$$

- (a) Give a context-free grammar G for L . Be sure to specify G as a 4-tuple $G = (V, \Sigma, R, S)$.

- (b) Give a PDA for L . You only need to draw the state diagram (graph).

Scratch-work area

5. [15 points] Recall the pumping lemma for regular languages:

Theorem: If L is a regular language, then there exists a pumping length p where, if $s \in L$ with $|s| \geq p$, then s can be split into three pieces $s = xyz$ such that (i) $xy^iz \in L$ for each $i \geq 0$, (ii) $|y| \geq 1$, and (iii) $|xy| \leq p$.

For $\Sigma = \{d, h, u\}$ (note the alphabet), let

$$A = \{ h^i u^j d^k \mid i, j, k \geq 0 \text{ and } i = j + k \}.$$

Is A a regular or nonregular language? If A is regular, give a regular expression **and** DFA (only state diagram) for A . If A is not regular, prove that it is a nonregular language.

Circle one:

Regular Language

Nonregular Language