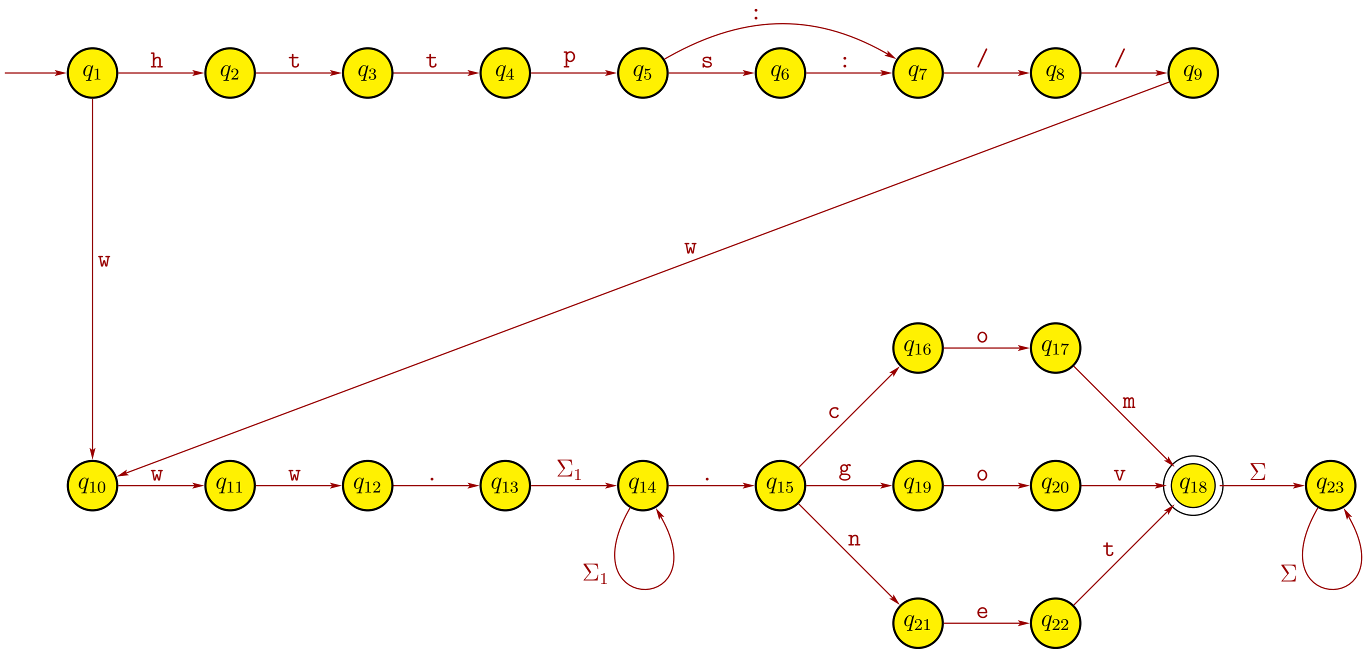
CS 341, Fall 2010
Solutions for Midterm, eLearning Section

1. (a) True. The language $\emptyset$ is finite, so slide 1-81 shows that it is regular. Corollary 2.32 then implies that $\emptyset$ is also context-free.
   (b) False. For example, let A have regular expression $(0 \cup 1)^*$, so it is an infinite language. Since A has a regular expression, it is a regular language by Theorem 1.54.
   (c) True. By Corollary 1.40, A is regular since it has an NFA. Corollary 2.32 then implies that A is context-free, so it has a PDA by Theorem 2.20.
   (d) False. The language $A = \{a^n b^n c^n \mid n \geq 0\}$ is nonregular, which we can show by the pumping lemma for regular languages (choose the string $a^p b^p c^p$ to get a contradiction). But slide 2-105 shows that A is also non-context-free.
   (e) False. Let $A = \{0^n 1^n \mid n \geq 0\}$ and B have regular expression $(0 \cup 1)^*$. Then B is regular since it has a regular expression (Theorem 1.54). Also, note that $A \subseteq B$, but A is nonregular, as shown on slide 1-90.
   (f) False. Homework 6, problem 2b.
   (g) True. Since A is finite, it is regular by slide 1-81. Thus, $\overline{A}$ is regular by Homework 2, problem 3. Also, B is regular since it has a regular expression (Theorem 1.54), so $\overline{A} \cap B$ is regular by slide 1-28. Hence, Corollary 2.32 implies $\overline{A} \cap B$ is context-free.
   (h) False. The derivation $S \Rightarrow 0$ generates the string 0, which is not in the language, so the CFG cannot be correct.
   (i) True. Homework 5, problem 3b.
   (j) True. Since A has a PDA, it is context-free by Theorem 2.20, so the statement then follows from Theorem 2.9.

2. (a) $1^*0(1 \cup 01^*0)^*$
   (b)
   - $S \to Y$ is not in Chomsky normal form since unit rules are not allowed.
   - $S \to ba$ is improper since a rule cannot have more than one terminal on the RHS.
   - $X \to XS$ is improper since starting variable S cannot be on RHS of rule.
   - $X \to \varepsilon$ is improper since ε cannot be on the RHS of rule when the left side is not S.
   - $Y \to XXY$ is improper since the RHS cannot have more than two variables.

   (c) slide 1-50.
   (d) Homework 5, problem 3a.

3. Below is a DFA for the language L. There are other correct DFAs for L.

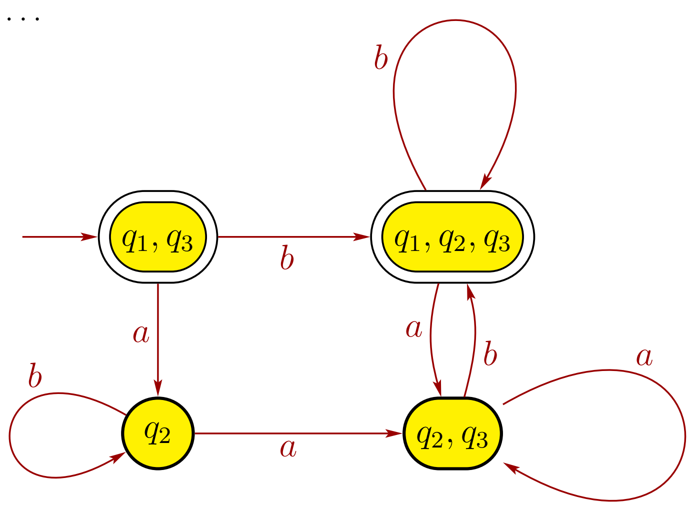


All edges not specified go to state q_{23}. For example, there is an edge from q_1 to q_{23} labeled $\Sigma - \{\mathtt{h}, \mathtt{w}\}$.

4. Homework 6, problem 2a.

5. (a) $\varepsilon, b, bb, aab, bab, \ldots$

   (b)



6. (a) $G = (V, \Sigma, R, S)$, with $V = \{S\}$, $\Sigma = \{a, b\}$, start variable S and rules $S \rightarrow aSa \mid bSb \mid \varepsilon$.

   (b) slide 2-59.

7. Suppose that A is a regular language. Let p be the pumping length, and consider the string $s = a^p b^{p+1} \in A$. Note that $|s| = 2p + 1 \geq p$, so the pumping lemma implies we can write $s = xyz$ with $xy^i z \in A$ for all $i \geq 0$, $|y| > 0$, and $|xy| \leq p$. Now, $|xy| \leq p$ implies that x and y have only a's (together up to p in total) and z has the rest of the a's at the beginning, followed by b^{p+1}. Hence, we can write $x = a^j$ for some $j \geq 0$, $y = a^k$ for some $k \geq 0$, and $z = a^\ell b^{p+1}$, where $j + k + \ell = p$ since $xyz = s = a^p b^{p+1}$. Also, $|y| > 0$ implies $k > 0$. Now consider the string $xyyz = a^j a^k a^k a^\ell b^{p+1} = a^{p+k} b^{p+1}$ since $j + k + \ell = p$. Note that $xyyz \notin A$ since $k \geq 1$ so the number of b's in the string is not greater than the number of a's. This contradicts (i), so A is not a regular language.