Midterm Exam
CS 341-451: Foundations of Computer Science II — **Fall 2010, eLearning section**
Prof. Marvin K. Nakayama

Print family (or last) name: ____________________________________________

Print given (or first) name: ____________________________________________

I have read and understand all of the instructions below, and I will obey the Academic Honor Code.

____________________________________________
Signature and Date

- This exam has 9 pages in total, numbered 1 to 9. Make sure your exam has all the pages.
- Unless other arrangements have been made with the professor, the exam is to last 2.5 hours and is to be given on Saturday, October 16, 2010.
- This is a closed-book, closed-note exam. No calculators are allowed.
- For all problems, follow these instructions:
  1. Give only your answers in the spaces provided. I will only grade what you put in the answer space, and I will take off points for any scratch work in the answer space. Use the scratch-work area or the backs of the sheets to work out your answers before filling in the answer space.
  2. DFA stands for deterministic finite automaton; NFA stands for nondeterministic finite automaton; CFG stands for context-free grammar; PDA stands for pushdown automaton.
  3. For any proofs, be sure to provide a step-by-step argument, with justifications for every step. If you are asked to prove a result X, in your proof of X, you may use any other result Y without proving Y. However, make it clear what the other result Y is that you are using; e.g., write something like, "By the result that $A^{**} = A^*$, we know that ...."

| Problem | 1 | 2 | 3 | 4 | 5 | 6 | 7 | Total |
|---|---|---|---|---|---|---|---|---|
| Points | | | | | | | | |

1. **[20 points]** For each of the following, circle TRUE if the statement is correct. Otherwise, circle FALSE

   (a) TRUE FALSE — $\emptyset$ is a context-free language.

   (b) TRUE FALSE — If $A$ is a regular language, then $A$ is finite.

   (c) TRUE FALSE — If $A$ has an NFA, then $A$ must have a PDA.

   (d) TRUE FALSE — Every nonregular language is context-free.

   (e) TRUE FALSE — If $A$ and $B$ are languages such that $A \subseteq B$ and $B$ is regular, then $A$ must be regular.

   (f) TRUE FALSE — The class of context-free languages is closed under complements.

   (g) TRUE FALSE — If $A$ is a finite language and $B$ has a regular expression, then $\overline{A} \cap B$ must be context-free.

   (h) TRUE FALSE — The language $\{ 1^n 0^n \mid n \geq 0 \}$ has context-free grammar $G = (V, \Sigma, R, S)$, with $V = \{S\}$, $\Sigma = \{0, 1\}$, start variable $S$, and rules $S \rightarrow 1S0 \mid 0$.

   (i) TRUE FALSE — The class of context-free languages is closed under concatenation.

   (j) TRUE FALSE — If $A$ has a PDA, then $A$ must have a CFG in Chomsky normal form.

2. **[20 points]** Give short answers to each of the following parts. **Each answer should be at most a few sentences. Be sure to define any notation that you use.**

(a) Let $\Sigma = \{0, 1\}$, and let $A$ be the set of strings over $\Sigma$ having an odd number of 0's. Give a regular expression for $A$.

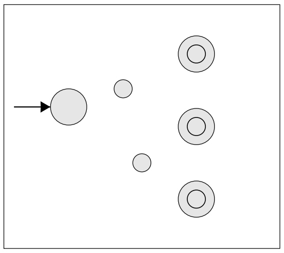
(b) Consider the following CFG $G = (V, \Sigma, R, S)$, with $V = \{S, X, Y\}$, $\Sigma = \{a, b\}$, start variable $S$, and rules $R$ as follows:

$$\begin{aligned} S &\rightarrow Y \mid XX \mid ba \mid \varepsilon \\ X &\rightarrow XS \mid \varepsilon \\ Y &\rightarrow a \mid XXY \end{aligned}$$

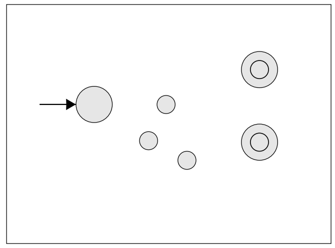
Note that $G$ is not in Chomsky normal form. List all of the rules in $G$ that violate Chomsky normal form. Explain your answer.

(c) Suppose that language $A_1$ is recognized by NFA $N_1$ below, and language $A_2$ is recognized by NFA $N_2$ below. Note that the transitions are not drawn in $N_1$ and $N_2$. Draw a picture of an NFA for $A_1 \circ A_2$.

$N_1$



$N_2$



(d) Suppose that $A_1$ is a language defined by a CFG $G_1 = (V_1, \Sigma, R_1, S_1)$, and $A_2$ is a language defined by a CFG $G_2 = (V_2, \Sigma, R_2, S_2)$, where the alphabet $\Sigma$ is the same for both languages and $V_1 \cap V_2 = \emptyset$. Let $A_3 = A_1 \cup A_2$. Give a CFG $G_3$ for $A_3$ in terms of $G_1$ and $G_2$. You do not have to prove the correctness of your CFG $G_3$, but do not give just an example.

3. **[10 points]** Define $\Sigma_1 = \{\texttt{a}, \texttt{b}, \texttt{c}, \ldots, \texttt{z}\}$ be the set of lower-case Roman letters, and let $\Sigma_2 = \{\texttt{.}, \texttt{:}, \texttt{/}\}$ be the set consisting of dot (`.`), colon (`:`) and slash (`/`). Let $\Sigma = \Sigma_1 \cup \Sigma_2$. Define the following sets:

   - $S_1 = \{\texttt{http://}, \texttt{https://}\}$
   - $S_2 = \{\texttt{www}\}$
   - $S_3 = \Sigma_1 \Sigma_1^*$, which consists of strings over $\Sigma_1$ of positive length
   - $S_4 = \{\texttt{com}, \texttt{gov}, \texttt{net}\}$.

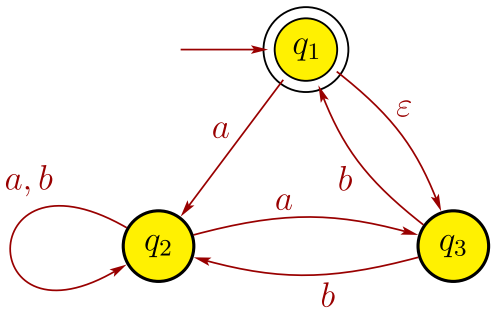
   Then we define the following sets of strings over $\Sigma$:

   - $L_1 = S_1 S_2 . S_3 . S_4$
   - $L_2 = S_2 . S_3 . S_4$
   - $L = L_1 \cup L_2$

   Give a DFA for the language $L$ with alphabet $\Sigma$. You only need to draw the graph; do not specify the DFA as a 5-tuple.

4. [**10 points**] Give an example of context-free languages $A$ and $B$ such that $C = A \cap B$ is not context-free. Explain your answer. Be sure to give CFGs for $A$ and $B$. You do not have to prove that $C$ is non-context-free for your example, but $C$ must be a non-context-free language that we went over in the course.

5. **[15 points]** Let $N$ be the following NFA with $\Sigma = \{a, b\}$, and let $C = L(N)$.




(a) List the strings in $C$ in lexicographic order. If $C$ has more than 5 strings, list only the first 5 strings in $C$, followed by 3 dots.

(b) Give a DFA for $C$.

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**Scratch-work area**

6. **[15 points]** Let $\Sigma = \{a, b\}$, and consider the language $A = \{\, w \in \Sigma^* \mid w = w^{\mathcal{R}}, |w| \text{ is even } \}$, where $w^{\mathcal{R}}$ denotes the reverse of $w$ and $|w|$ denotes the length of $w$.

   (a) Give a CFG $G$ for $A$. Be sure to specify $G$ as a 4-tuple $G = (V, \Sigma, R, S)$.

   (b) Give a PDA for $A$. You only need to give the drawing.

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**Scratch-work area**

7. **[10 points]** Recall the pumping lemma for regular languages:

**Theorem:** If $L$ is a regular language, then there is a number $p$ (pumping length) where, if $s \in L$ with $|s| \geq p$, then there are strings $x$, $y$, $z$ such that $s = xyz$ and

**(i)** $xy^iz \in L$ for each $i \geq 0$,

**(ii)** $|y| > 0$, and

**(iii)** $|xy| \leq p$.

Let $\Sigma = \{a, b\}$, and consider the language $A = \{\, w \in \Sigma^* \mid w \text{ has more } b\text{'s than } a\text{'s} \,\}$. Prove that $A$ is not a regular language.