Midterm Exam 1 CS 341: Foundations of Computer Science II — **Spring 2010, day section** Prof. Marvin K. Nakayama

Print family (or last) name:

Print given (or first) name:

I have read and understand all of the instructions below, and I will obey the Academic Honor Code.

Signature and Date

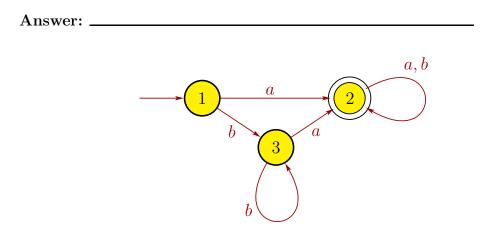
- This exam has 7 pages in total, numbered 1 to 7. Make sure your exam has all the pages.
- Note the number written on the upper right-hand corner of the first page. On the sign-up sheet being passed around, sign your name next to this number.
- This exam will be 1 hour and 25 minutes in length.
- This is a closed-book, closed-note exam.
- For all problems, follow these instructions:
 - 1. Give only your answers in the spaces provided. I will only grade what you put in the answer space, and I will take off points for any scratch work in the answer space. Use the scratch-work area or the backs of the sheets to work out your answers before filling in the answer space.
 - 2. For any proofs, be sure to provide a step-by-step argument, with justifications for every step.

Problem	1	2	3	4	5	Total
Points						

1. **[20 points]** For each of the following, circle TRUE if the statement is correct. Otherwise, circle FALSE

(a)	TRUE	FALSE		If $A \subseteq B$ and B is a regular language, then A must be regular.
(b)	TRUE	FALSE		If $A \subseteq B$ and A is a regular language, then B must be regular.
(c)	TRUE	FALSE	_	If A is regular, then A must be finite.
(d)	TRUE	FALSE		Every context-free language is also regular.
(e)	TRUE	FALSE		The class of regular languages is closed under inter- section.
(f)	TRUE	FALSE		A regular expression for $A = \{ 0^n 1^n \mid n \ge 0 \}$ is $0^* 1^*$.
(g)	TRUE	FALSE		If A has an NFA, then A is nonregular.
(h)	TRUE	FALSE		If A has a DFA, then A must have a context-free grammar.
(i)	TRUE	FALSE		If a language A has a PDA, then A must have a context-free grammar in Chomsky normal form.
(j)	TRUE	FALSE		The transition function of an NFA is $\delta: Q \times \Sigma_{\varepsilon} \to Q$.

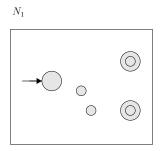
- 2. **[20 points]** Give short answers to each of the following parts. Be sure to define any notation that you use.
 - (a) Give a regular expression for the language recognized by the DFA below.



(b) For the alphabet $\Sigma = \{a, b\}$, give a regular expression for the language of strings having an odd number of b's.

Answer: _____

(c) Suppose that language A_1 is recognized by NFA N_1 below. Note that the transitions are not drawn in N_1 . Draw a picture of an NFA for A_1^* .

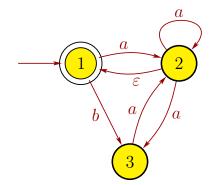


(d) Consider the following CFG $G = (V, \Sigma, R, S)$, with $V = \{S, X, Y\}$, $\Sigma = \{a, b\}$, start variable S, and rules R as follows:

$$\begin{array}{rcl} S & \to & ba \mid Yb \mid \varepsilon \\ X & \to & YS \mid \varepsilon \\ Y & \to & a \mid X \end{array}$$

Note that G is not in Chomsky normal form. List all of the rules in G that violate Chomsky normal form. Explain your answer.

3. [20 points] Let N be the following NFA with $\Sigma = \{a, b\}$, and let C = L(N).



- (a) List the strings in C in lexicographic order. If C has more than 5 strings, list only the first 5 strings in C, followed by 3 dots.
- (b) Give a DFA for C.

Scratch-work area

4. **[25 points]** Consider the alphabet $\Sigma = \{a, b, c\}$ and the language

$$L = \{ b^{i} c^{j} a^{k} \mid i, j, k \ge 0 \text{ and } i + j = k \}.$$

(a) Give a context-free grammar G for L. Be sure to specify G as a 4-tuple $G = (V, \Sigma, R, S)$.

(b) Give a PDA for L. You only need to draw the graph.

Scratch-work area

5. **[15 points]** Recall the pumping lemma for regular languages:

Theorem: If L is a regular language, then there exists a pumping length p where, if $s \in L$ with $|s| \geq p$, then there exists strings x, y, z such that s = xyz and (i) $xy^i z \in L$ for each $i \geq 0$, (ii) $|y| \geq 1$, and (iii) $|xy| \leq p$.

Let $A = \{ b^i c^j a^k \mid i, j, k \ge 0 \text{ and } i + j = k \}$. Is A a regular or nonregular language? If A is regular, give a regular expression for A. If A is not regular, prove that it is a nonregular language.

Circle one: Regular Language Nonregular Language