CS 341, Spring 2010
Solutions for Midterm 2

1. (a) True. This is just the definition of co-Turing-recognizable.
   (b) False, by Theorem 3.16.
   (c) False. A TM M may loop on input w.
   (d) False. $\overline{A_{\mathrm{TM}}}$ is not Turing-recognizable by Corollary 4.23.
   (e) True, since the definition of Turing-decidable is more restrictive than the definition of Turing-recognizable.
   (f) True, by Theorem 3.13.
   (g) True, by slide 4-25.
   (h) False, e.g., if $A = \{00, 11\}$ and $B = \{00, 11, 111\}$, then $A \cap \overline{B} = \emptyset$ but $A \neq B$. For A and B to be equal, we instead need $(\overline{A} \cap B) \cup (A \cap \overline{B}) = \emptyset$.
   (i) False, since the set $\mathcal{N} = \{1, 2, 3, \ldots\}$ is countable.
   (j) True, since every regular language is context-free by Corollary 2.32, and every context-free language is decidable by Theorem 4.9.

2. (a) Yes, because each element in A maps to a different element in B.
   (b) No, because there is no element in A that maps to $4 \in B$.
   (c) No, because f is not onto.
   (d) An algorithm is a Turing machine that always halts.
   (e) A language L_1 that is Turing-recognizable has a Turing machine M_1 such that M_1 accepts each $w \in L_1$, and M_1 loops or rejects every $w \notin L_1$. A language L_2 that is Turing-decidable has a Turing machine M_2 such that M_2 accepts each $w \in L_2$, and M_2 rejects every $w \notin L_2$; i.e., M_2 never loops.

3. (a) $q_1 110\#01$ $\quad xq_3 10\#01$ $\quad x1q_3 0\#01$ $\quad x10q_3\#01$ $\quad x10\#q_5 01$ $\quad x10\#0q_{\text{reject}}1$
   (b) $q_1 0\#0$ $\quad xq_2\#0$ $\quad x\#q_4 0$ $\quad xq_6\#x$ $\quad q_7 x\#x$ $\quad xq_1\#x$ $\quad x\#q_8 x$ $\quad x\#xq_8$ $\quad x\#x\sqcup q_{\text{accept}}$

4. Slides 4-39 and 4-40.

5. Define the language as

$$C = \{\langle D, R\rangle \mid D \text{ is a DFA and } R \text{ is a regular expression with } L(D) = L(R)\}.$$

Recall that the proof of Theorem 4.5 defines a Turing machine F that decides the language $EQ_{\text{DFA}} = \{\langle A, B\rangle \mid A \text{ and } B \text{ are DFAs and } L(A) = L(B)\}$. Then the following Turing machine T decides C:

T = "On input $\langle D, R\rangle$, where D is a DFA and R is a regular expression:

1. Convert R into an equivalent DFA D' using the algorithm in the proof of Kleene's Theorem.

2. Run TM F from Theorem 4.5 on input $\langle D, D'\rangle$.

3. If F accepts, *accept*. If F rejects, *reject*."

6. Homework 8, problem 4.