Midterm Exam
CS 341-451: Foundations of Computer Science II — **Fall 2011, eLearning section**
Prof. Marvin K. Nakayama

Print family (or last) name: ____

Print given (or first) name: ____

I have read and understand all of the instructions below, and I will obey the Academic Honor Code.

____
Signature and Date

- This exam has 9 pages in total, numbered 1 to 9. Make sure your exam has all the pages.
- Unless other prior arrangements have been made with the professor, the exam is to last 2.5 hours and is to be given on Saturday, October 22, 2011.
- This is a closed-book, closed-note exam. No calculators are allowed.
- For all problems, follow these instructions:
  1. Give only your answers in the spaces provided. I will only grade what you put in the answer space, and I will take off points for any scratch work in the answer space. Use the scratch-work area or the backs of the sheets to work out your answers before filling in the answer space.
  2. DFA stands for deterministic finite automaton; NFA stands for nondeterministic finite automaton; CFG stands for context-free grammar; PDA stands for pushdown automaton. TM stands for Turing machine.
  3. For any proofs, be sure to provide a step-by-step argument, with justifications for every step. If you are asked to prove a result X, in your proof of X, you may use any other result Y without proving Y. However, make it clear what the other result Y is that you are using; e.g., write something like, "By the result that $A^{**} = A^*$, we know that ...."

Problem	1	2	3	4	5	6	7	Total
Points								

1. **[20 points]** For each of the following, circle TRUE if the statement is correct. Otherwise, circle FALSE

(a) TRUE FALSE — If A and B are regular languages, then so is $A^* \cap \overline{B}$.

(b) TRUE FALSE — If A has a regular expression, then A has a PDA.

(c) TRUE FALSE — If $A \subseteq B$ and A is a regular language, then B is a regular language.

(d) TRUE FALSE — There is a language recognized by an NFA but has no DFA.

(e) TRUE FALSE — If A is a context-free language that is also non-regular, then A has a CFG in Chomsky normal form.

(f) TRUE FALSE — Every non-context-free language is also non-regular.

(g) TRUE FALSE — If A is a regular language, then A is finite.

(h) TRUE FALSE — If $M = (Q, \Sigma, \Gamma, \delta, q_0, q_{\text{accept}}, q_{\text{reject}})$ is a Turing machine and $w \in \Sigma^*$ is a string, then M either accepts or rejects w.

(i) TRUE FALSE — Every nonregular language is context-free.

(j) TRUE FALSE — The class of context-free languages is closed under intersection.

2. **[20 points]** Give short answers to each of the following parts. **Each answer should be at most a few sentences. Be sure to define any notation that you use.**

   (a) Let $\Sigma = \{a, b\}$, and let A be the set of strings that have exactly two a's or at least two b's. Give a regular expression for A.

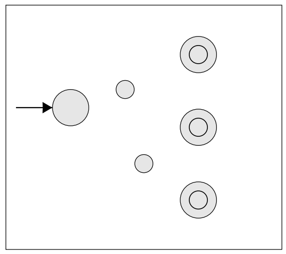
   (b) Consider the following CFG $G = (V, \Sigma, R, S)$, with $V = \{S, X, Y\}$, $\Sigma = \{a, b\}$, start variable S, and rules R as follows:

$$\begin{array}{rcl} S & \rightarrow & YS \mid XY \mid b \mid \varepsilon \\ X & \rightarrow & YX \mid \varepsilon \\ Y & \rightarrow & aX \mid a \mid XYX \end{array}$$

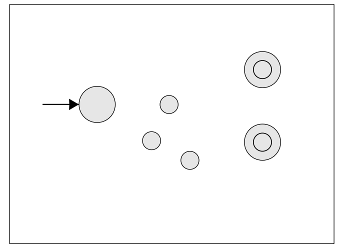
   Note that G is not in Chomsky normal form. List all of the rules in G that violate Chomsky normal form. Explain your answer.

(c) Suppose that language A_1 is recognized by NFA N_1 below, and language A_2 is recognized by NFA N_2 below. Note that the transitions are not drawn in N_1 and N_2. Draw a picture of an NFA for $A_1 \circ A_2$.

N_1



N_2



(d) Suppose that A is a language defined by a CFG $G_1 = (V_1, \Sigma, R_1, S_1)$. Give a CFG G_2 for A^* in terms of G_1. You do not have to prove the correctness of your CFG G_2, but do not give just an example.

3. [**10 points**] Define $\Gamma = \{\texttt{a}, \texttt{b}, \texttt{c}, \ldots, \texttt{z}\}$ be the set of lower-case Roman letters. Let $\Sigma = \Gamma \cup \{.\}$ be the alphabet of lower-case Roman letters and the dot. Define the following sets:

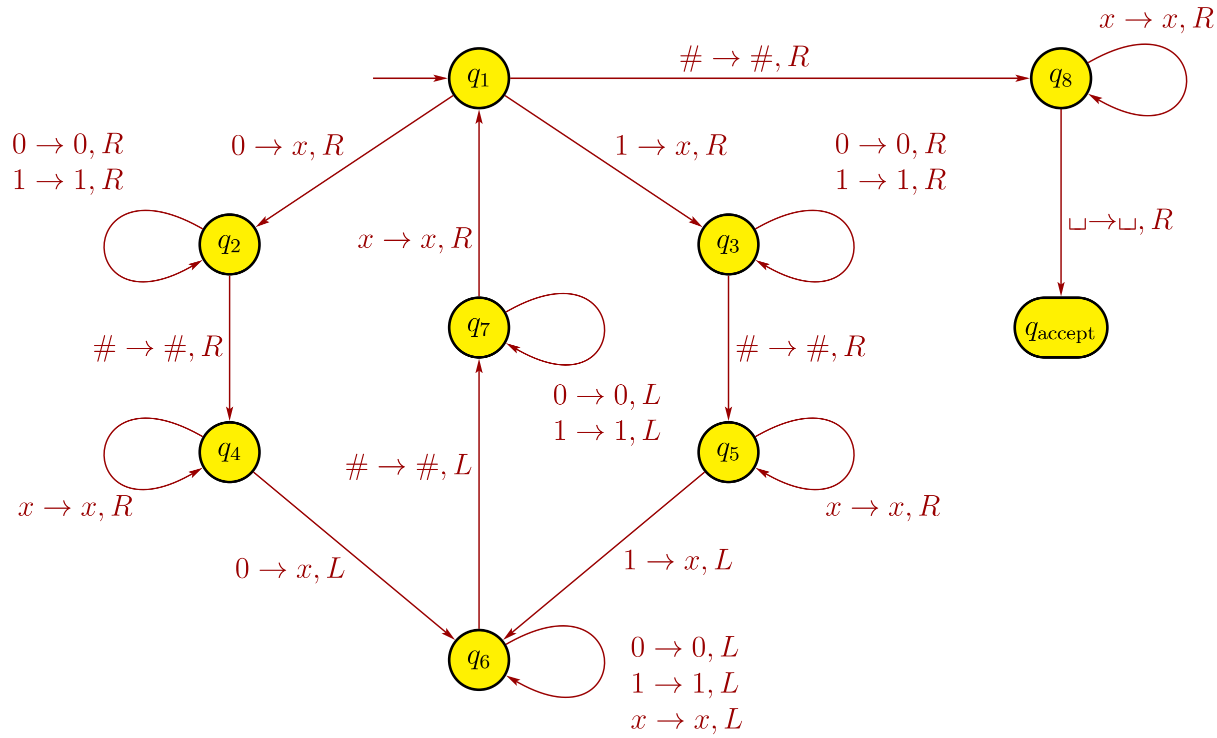
   - $S_1 = \{\texttt{www}\}$
   - $S_2 = \Gamma\Gamma^*$
   - $S_3 = \{\texttt{com}, \texttt{net}\}$

   Then we define

   $$L = S_1.S_2.S_3$$

   as a certain set of web addresses. Give a DFA for the language L with alphabet Σ. You only need to draw the graph; do not specify the DFA as a 5-tuple. You must specify *all* transitions. To simplify your drawing, you may use the notation $\Gamma_\ell = \Gamma - \{\ell\}$ for any symbol $\ell \in \Gamma$.

4. **[10 points]** Consider the below Turing machine $M = (Q, \Sigma, \Gamma, \delta, q_1, q_{\text{accept}}, q_{\text{reject}})$ with $Q = \{q_1, \ldots, q_8, q_{\text{accept}}, q_{\text{reject}}\}$, $\Sigma = \{0, 1, \#\}$, $\Gamma = \{0, 1, \#, x, \sqcup\}$, and transitions below.



To simplify the figure, we don't show the reject state q_{reject} or the transitions going to the reject state. Those transitions occur implicitly whenever a state lacks an outgoing transition for a particular symbol. For example, because in state q_5 no outgoing arrow with a # is present, if a # occurs under the head when the machine is in state q_5, it goes to state q_{reject}. For completeness, we say that in each of these transitions to the reject state, the head writes the same symbol as is read and moves right.

In each of the parts below, give the sequence of configurations that M enters when started on the indicated input string.

(a) 110#01

(b) 0#0

5. **[20 points]** Let $\Sigma = \{a, b, c\}$, and consider the language $A = \{a^n b^n c^k \mid n, k \geq 0\}$.

(a) Give a CFG G for A. Be sure to specify G as a 4-tuple $G = (V, \Sigma, R, S)$.

(b) Give a PDA for A. You only need to give the drawing.

Scratch-work area

Each of the following problems requires you to prove a result. Unless stated otherwise, in your proofs, you can apply any theorems or results that we went over in class (lectures, homeworks) without proving them, except for the result you are asked to prove in the problem. When citing a theorem or result, make sure that you give enough details so that it is clear what theorem or result you are using (e.g., say something like, "By the result that says $A^{**} = A^*$, we can show that ....")

6. **[10 points]** Show that the class of context-free languages is not closed under complementation. [Hint: consider the language in problem 5 and use DeMorgan's law $A \cap B = \overline{\overline{A} \cup \overline{B}}$.]

7. **[10 points]** Recall the pumping lemma for regular languages:

Theorem: If L is a regular language, then there is a number p (pumping length) where, if $s \in L$ with $|s| \geq p$, then there are strings x, y, z such that $s = xyz$ and

(i) $xy^i z \in L$ for each $i \geq 0$,

(ii) $|y| > 0$, and

(iii) $|xy| \leq p$.

Let $\Sigma = \{a, b, c\}$, and consider the language $A = \{a^n b^n c^k \mid n, k \geq 0\}$. Prove that A is not a regular language.