Midterm Exam 1
CS 341: Foundations of Computer Science II — **Fall 2011, day section**
Prof. Marvin K. Nakayama

Print family (or last) name: ________________________________

Print given (or first) name: ________________________________

I have read and understand all of the instructions below, and I will obey the Academic Honor Code.

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Signature and Date

- This exam has 7 pages in total, numbered 1 to 7. Make sure your exam has all the pages.
- Note the number written on the upper right-hand corner of the first page. On the sign-up sheet being passed around, sign your name next to this number.
- This exam will be 1 hour and 25 minutes in length.
- This is a closed-book, closed-note exam.
- For all problems, follow these instructions:
  1. Give only your answers in the spaces provided. I will only grade what you put in the answer space, and I will take off points for any scratch work in the answer space. Use the scratch-work area or the backs of the sheets to work out your answers before filling in the answer space.
  2. For any proofs, be sure to provide a step-by-step argument, with justifications for every step.

Problem	1	2	3	4	5	Total
Points						

1. **[20 points]** For each of the following, circle TRUE if the statement is correct. Otherwise, circle FALSE

(a) TRUE FALSE — If A is a regular language, then A is finite.

(b) TRUE FALSE — The language $A = \{\, 0^n 1^n \mid 0 \le n \le 53 \,\}$ is regular.

(c) TRUE FALSE — Every nonregular language is context-free.

(d) TRUE FALSE — The class of context-free languages is closed under complementation.

(e) TRUE FALSE — If A and B are regular languages, then so is $\overline{A} \cup B^*$.

(f) TRUE FALSE — If A has a regular expression, then A has a PDA.

(g) TRUE FALSE — Every subset of a regular language is also regular.

(h) TRUE FALSE — There is a language recognized by an NFA but has no DFA.

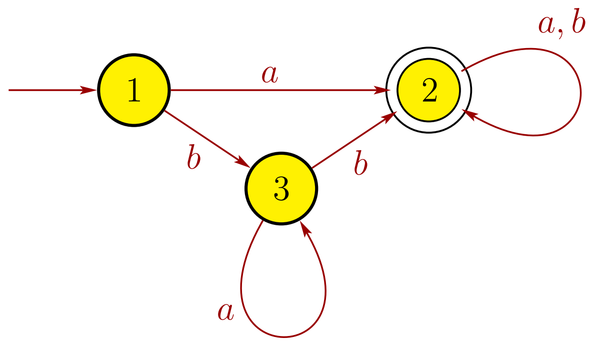
(i) TRUE FALSE — If A is a context-free language that is also non-regular, then A has a CFG in Chomsky normal form.

(j) TRUE FALSE — Every non-context-free language is also non-regular.

2. **[20 points]** Give short answers to each of the following parts. Be sure to define any notation that you use.

(a) Give a regular expression for the language recognized by the DFA below.

Answer: ____



(b) For the alphabet $\Sigma = \{a, b\}$, give a regular expression for the language

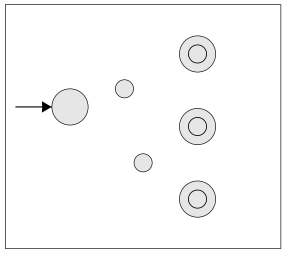
$$\{w \in \Sigma^* \mid w \text{ does } \textit{not} \text{ end in a double letter}\}.$$

(A string contains a *double letter* if it contains aa or bb as a substring.)

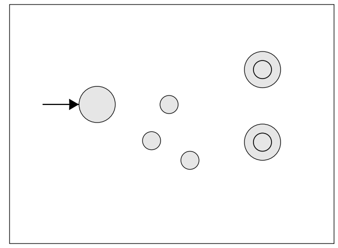
Answer: ____

(c) Suppose that language A_1 is recognized by NFA N_1 below, and language A_2 is recognized by NFA N_2 below. Note that the transitions are not drawn in N_1 and N_2. Draw a picture of an NFA for $A_1 \circ A_2$.

N_1



N_2

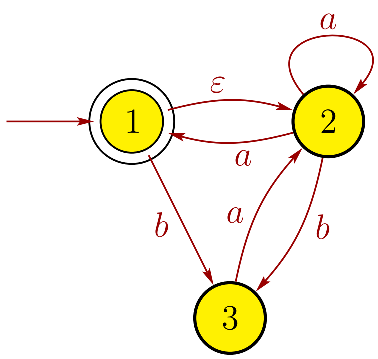


(d) Consider the following CFG $G = (V, \Sigma, R, S)$, with $V = \{S, X, Y\}$, $\Sigma = \{a, b\}$, start variable S, and rules R as follows:

$$\begin{aligned} S &\rightarrow XY \mid aX \mid \varepsilon \\ X &\rightarrow Y \mid b \\ Y &\rightarrow \varepsilon \mid SX \end{aligned}$$

Note that G is not in Chomsky normal form. List all of the rules in G that violate Chomsky normal form. Explain your answer.

3. **[20 points]** Let N be the following NFA with $\Sigma = \{a, b\}$, and let $C = L(N)$.



(a) List the strings in C in lexicographic order. If C has more than 5 strings, list only the first 5 strings in C, followed by 3 dots.

(b) Give a DFA for C.

Scratch-work area

4. **[25 points]** Consider the alphabet $\Sigma = \{a, b, c\}$ and the language

$$L = \{ \, c^i a^j b^k \mid i, j, k \geq 0, \text{ and } i = j \text{ or } i = k \, \}.$$

(a) Give a context-free grammar G for L. Be sure to specify G as a 4-tuple $G = (V, \Sigma, R, S)$.

(b) Give a PDA for L. You only need to draw the graph.

Scratch-work area

5. **[15 points]** Recall the pumping lemma for regular languages:

Theorem: If L is a regular language, then there exists a pumping length p where, if $s \in L$ with $|s| \geq p$, then there exists strings x, y, z such that $s = xyz$ and (i) $xy^iz \in L$ for each $i \geq 0$, (ii) $|y| \geq 1$, and (iii) $|xy| \leq p$.

Let $A = \{ www \mid w \in \{a, b\}^* \}$. Is A a regular or nonregular language? If A is regular, give a regular expression for A. If A is not regular, prove that it is a nonregular language.

Circle one: **Regular Language** **Nonregular Language**