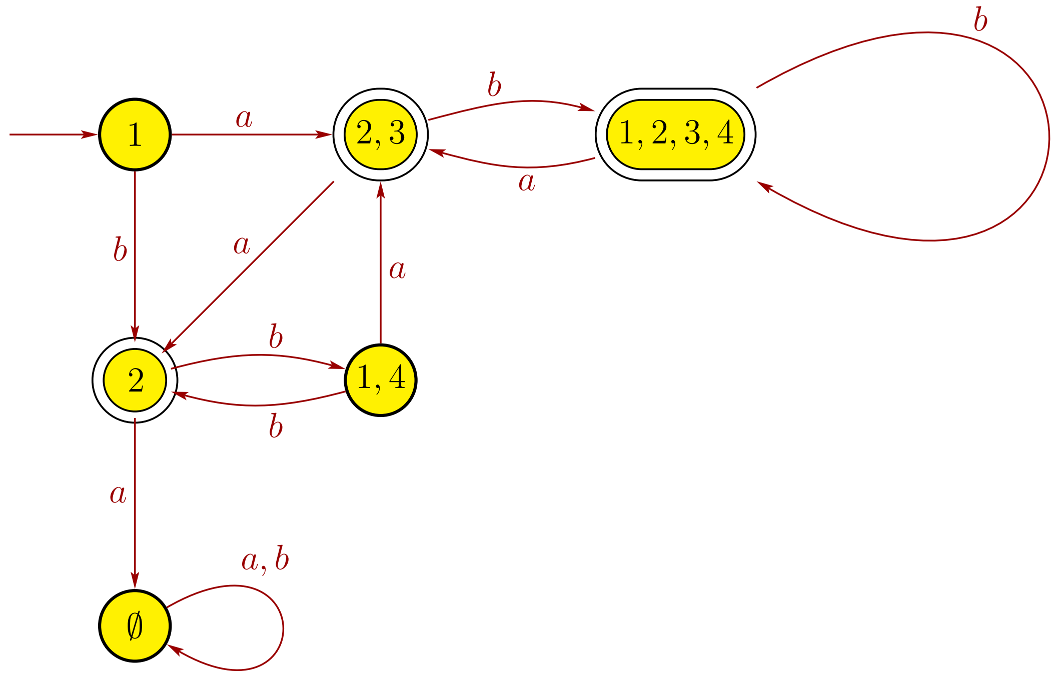
CS 341, Fall 2012
Solutions for Midterm, eLearning Section

1. (a) False. The language a^* is regular but infinite.
   (b) True. Since A is finite, it is regular by slide 1-81 of the notes. Corollary 2.32 then ensures that A is regular.
   (c) False. For example, let $A = \{a^n b^n \mid n \geq 0\}$ and $B = (a \cup b)^*$. Then $A \subseteq B$, B is regular, but A is nonregular.
   (d) False. a^*b^* generates the string $abb \notin \{a^n b^n \mid n \geq 0\}$.
   (e) True. By Theorem 2.9. The fact that A is non-regular is irrelevant.
   (f) True. Since A has an NFA, it is regular by Corollary 1.40. Since B is finite, it is regular by slide 1-81, and we know $\overline{B}$ is regular by HW 2, problem 3. Thus, $A \cap \overline{B}$ is regular by HW 2, problem 5.
   (g) True. By Theorem 2.20.
   (h) False. For example, let $A = \{abc\}$ and $B = \{a^n b^n c^n \mid n \geq 0\}$, so $A \subseteq B$. A is finite so it is regular, so it is also context-free by Corollary 2.32. But B is not context-free by slide 2-106.
   (i) False. By Homework 6, problem 2(b).
   (j) False. $A = \{a^n b^n c^n \mid n \geq 0\}$ is nonregular and not context-free.

2. (a) $\varepsilon \cup a \cup b \cup (a \cup b)^*(ab \cup ba)$
   (b)
   - $S \rightarrow X$ is improper since it is a unit rule.
   - $S \rightarrow ba$ is improper since a rule can't have more than one terminal on the right side.
   - $X \rightarrow YS$ is improper since the start variable S can't be on the right side.
   - $X \rightarrow \varepsilon$ is improper if ε is on the right side, S must be on the left side.
   - $Y \rightarrow aX$ is improper since the right side has a mix of terminals and variables.

   (c) slide 1-53.
   (d) Homework 5, problem 3a.

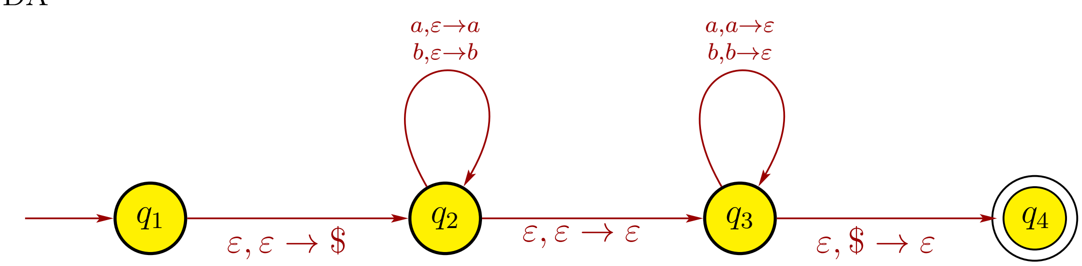
3. Here's a DFA for C.



4. (a) CFG $G = (V, \Sigma, R, S)$, with $V = \{S\}$ and start variable S, $\Sigma = \{a, b\}$, and rules R:

$$S \to aSa \mid bSb \mid \varepsilon$$

(b) PDA



5. HW 2, problem 4.

6. HW 6, problem 2b.

7. Suppose that A is a regular language. Let p be the pumping length, and consider the string $s = a^p bba^p \in A$. Note that $|s| = 2p + 2 \geq p$, so the pumping lemma implies we can write $s = xyz$ with $xy^iz \in A$ for all $i \geq 0$, $|y| > 0$, and $|xy| \leq p$. Now, $|xy| \leq p$ implies that x and y have only a's (together up to p in total) and z has the rest of the a's at the beginning, followed by bba^p. Hence, we can write $x = a^j$ for some $j \geq 0$, $y = a^k$ for some $k \geq 0$, and $z = a^\ell bba^p$, where $j + k + \ell = p$ since $xyz = s = a^p bba^p$. Also, $|y| > 0$ implies $k > 0$. Now consider the string $xyyz = a^j a^k a^k a^\ell bba^p = a^{p+k} bba^p$ since $j + k + \ell = p$. Since $k > 0$, the numbers of a's at the beginning and end of

$xyyz$ are not the same, so $xyyz$ is not the same forwards and backwards, which means $xyyz \notin A$. This contradicts (i), so A is not a regular language.