Midterm Exam
CS 341-451: Foundations of Computer Science II — **Fall 2012, eLearning section**
Prof. Marvin K. Nakayama

Print family (or last) name: ____________________________________

Print given (or first) name: ____________________________________

I have read and understand all of the instructions below, and I will obey the Academic Honor Code.

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Signature and Date

- This exam has 9 pages in total, numbered 1 to 9. Make sure your exam has all the pages.
- Unless other prior arrangements have been made with the professor, the exam is to last 2.5 hours and is to be given on Saturday, October 20, 2012.
- This is a closed-book, closed-note exam. No calculators are allowed.
- For all problems, follow these instructions:
  1. Give only your answers in the spaces provided. I will only grade what you put in the answer space, and I will take off points for any scratch work in the answer space. Use the scratch-work area or the backs of the sheets to work out your answers before filling in the answer space.
  2. DFA stands for deterministic finite automaton; NFA stands for nondeterministic finite automaton; CFG stands for context-free grammar; PDA stands for pushdown automaton. TM stands for Turing machine.
  3. For any proofs, be sure to provide a step-by-step argument, with justifications for every step. If you are asked to prove a result X, in your proof of X, you may use any other result Y without proving Y. However, make it clear what the other result Y is that you are using; e.g., write something like, "By the result that $A^{**} = A^*$, we know that . . . ."

Problem	1	2	3	4	5	6	7	Total
Points								

1. **[20 points]** For each of the following, circle TRUE if the statement is correct. Otherwise, circle FALSE

(a) TRUE FALSE — If A is a regular language, then A is finite.

(b) TRUE FALSE — If A is a finite language, then A is context-free.

(c) TRUE FALSE — If $A \subseteq B$ and B is a regular language, then A is a regular language.

(d) TRUE FALSE — The language $\{ a^n b^n \mid n \geq 0\}$ has regular expression $a^* b^*$.

(e) TRUE FALSE — If A is a context-free language that is also non-regular, then A has a CFG in Chomsky normal form.

(f) TRUE FALSE — If A has an NFA and B is a finite language, then $A \cap \overline{B}$ is regular.

(g) TRUE FALSE — If A has a context-free grammar, then A has a PDA.

(h) TRUE FALSE — If $A \subseteq B$ and A is a context-free language, then B is a context-free language.

(i) TRUE FALSE — The class of context-free languages is closed under complementation.

(j) TRUE FALSE — Every nonregular language is context-free.

2. **[20 points]** Give short answers to each of the following parts. **Each answer should be at most a few sentences. Be sure to define any notation that you use.**

   (a) Let $\Sigma = \{a, b\}$, and we say that a string $w \in \Sigma^*$ ends in a double letter if its last two symbols are aa or bb. Let $A = \{ \ w \in \Sigma^* \mid w \text{ does } \textbf{not} \text{ end in a double letter } \}$. Give a regular expression for A.

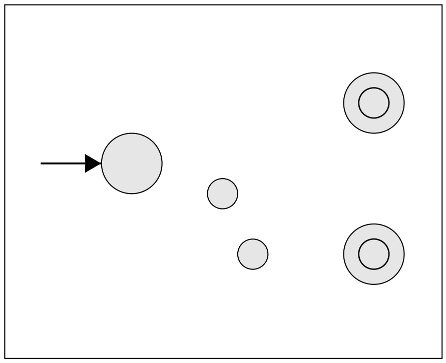
   (b) Consider the following CFG $G = (V, \Sigma, R, S)$, with $V = \{S, X, Y\}$, $\Sigma = \{a, b\}$, start variable S, and rules R as follows:

$$
\begin{aligned}
S &\rightarrow X \mid ba \mid \varepsilon \\
X &\rightarrow YS \mid \varepsilon \\
Y &\rightarrow aX \mid a \mid XY
\end{aligned}
$$

   Note that G is not in Chomsky normal form. List all of the rules in G that violate Chomsky normal form. Explain your answer.

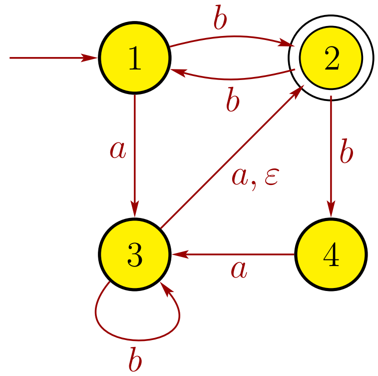
(c) Suppose that language A_1 is recognized by NFA N_1 below. Note that the transitions are not drawn in N_1. Draw a picture of an NFA for A_1^*.

N_1



(d) Suppose that A_1 is a language defined by a CFG $G_1 = (V_1, \Sigma, R_1, S_1)$, and A_2 is a language defined by a CFG $G_2 = (V_2, \Sigma, R_2, S_2)$, where the alphabet Σ is the same for both languages and $V_1 \cap V_2 = \emptyset$. Let $A_3 = A_1 \cup A_2$. Give a CFG G_3 for A_3 in terms of G_1 and G_2. You do not have to prove the correctness of your CFG G_3, but do not give just an example.

3. **[10 points]** Let N be the following NFA with $\Sigma = \{a, b\}$, and let $C = L(N)$.



Give a DFA for C.

4. **[20 points]** Let $\Sigma = \{a, b\}$, and consider the language $A = \{ w \in \Sigma^* \mid w = w^{\mathcal{R}} \text{ and } |w| \text{ is even } \}$.

(a) Give a CFG G for A. Be sure to specify G as a 4-tuple $G = (V, \Sigma, R, S)$.

(b) Give a PDA for A. You only need to give the drawing.

Scratch-work area

Each of the following problems may require you to prove a result. Unless stated otherwise, in your proofs, you can apply any theorems or results that we went over in class (lectures, homeworks) without proving them, except for the result you are asked to prove in the problem. When citing a theorem or result, make sure that you give enough details so that it is clear what theorem or result you are using (e.g., say something like, "By the result that says $A^{**} = A^*$, we can show that ....")

5. [**10 points**] We say that a DFA M for a language A is *minimal* if there does not exist another DFA M' for A such that M' has strictly fewer states than M. Suppose that $M = (Q, \Sigma, \delta, q_0, F)$ is a minimal DFA for A. Using M, we construct a DFA $\overline{M}$ for the complement $\overline{A}$ as $\overline{M} = (Q, \Sigma, \delta, q_0, Q - F)$. Prove that $\overline{M}$ is a minimal DFA for $\overline{A}$.

6. [**10 points**] Give an example of two context-free languages A and B such that their intersection C is not context-free. Be sure to give CFGs for A and B, and explain what C is. If we went over in class that C is not context-free, be sure to state this; if we didn't show your C is not context-free in class, then prove that C is not context-free.

7. **[10 points]** Recall the pumping lemma for regular languages:

Theorem: If L is a regular language, then there is a number p (pumping length) where, if $s \in L$ with $|s| \geq p$, then there are strings x, y, z such that $s = xyz$ and

(i) $xy^iz \in L$ for each $i \geq 0$,

(ii) $|y| > 0$, and

(iii) $|xy| \leq p$.

Let $\Sigma = \{a, b\}$, and consider the language $A = \{w \in \Sigma^* \mid w = w^{\mathcal{R}} \text{ and } |w| \text{ is even } \}$. Prove that A is not a regular language.