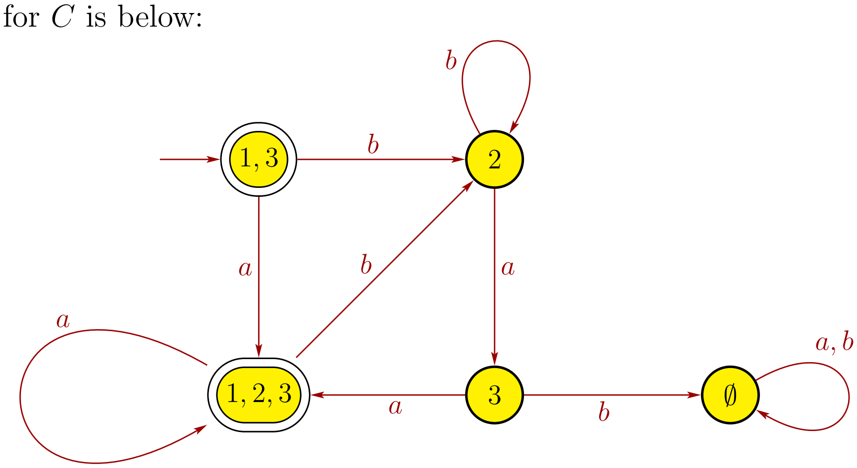
CS 341, Fall 2012, Face-to-Face Section
Solutions for Midterm 1

1. (a) True. By Corollary 1.40 and Theorem 1.54.
   (b) True. Homework 4, problem 5d.
   (c) True, by Lemma 2.27 and Theorem 2.9.
   (d) False. Let $A = \{0,1\}^*$ and $B = \{ww \mid w \in \{0,1\}^*\}$. Language A has CFG with rules $S \rightarrow 0S \mid 1S \mid \varepsilon$, so it is context-free. Slide 2-108 shows that B is not context-free.
   (e) True. Finite languages are regular by the theorem on slide 1-81.
   (f) False. The language $A = \{0^n1^n \mid n \geq 0\}$ is context-free since it has CFG with rules $S \rightarrow 0S1 \mid \varepsilon$, but it is non-regular, as shown on slide 1-90.
   (g) False. The language a^* is regular but infinite.
   (h) False. Corollary 1.40 shows that a language is regular if and only if it has an NFA.
   (i) False. Homework 6, problem 2(a).
   (j) True. If A has a regular expression, then it must be regular by Theorem 1.54. Homework 2, problem 3, shows that $\overline{A}$ must be regular. Corollary 2.32 implies that $\overline{A}$ must be context-free.

2. (a) $1^*0(1 \cup 01^*0)^*$
   (b) $(a \cup b(a \cup b)^*a)b^*$
   (c) slide 1-50.
   (d) Homework 5, problem 3a.

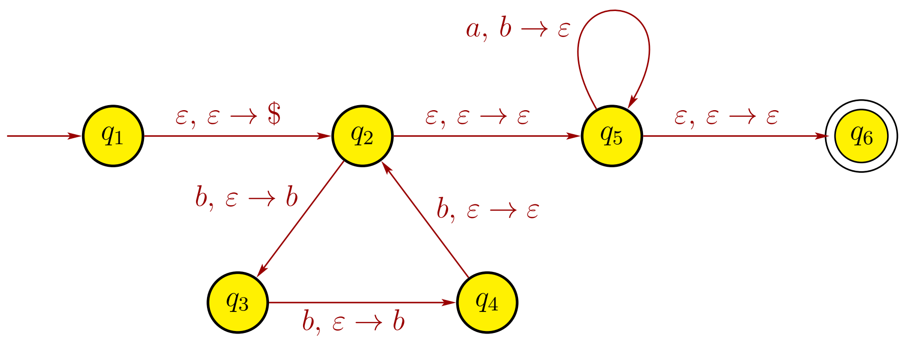
3. A DFA for C is below:



4. (a) $G = (V, \Sigma, R, S)$ with set of variables $V = \{S\}$, where S is the start variable; set of terminals $\Sigma = \{a, b\}$; and rules

$$S \rightarrow bbbSaa \mid \varepsilon$$

(b)



5. Language A is nonregular. We prove this by contradiction. Suppose that A is a regular language. Let p be the "pumping length" of the Pumping Lemma. Consider the string $s = a^{2p}c^{3p}b^{2p}$. Note that $s \in A$, and $|s| = 7p > p$, so the Pumping Lemma will hold. Thus, there exists strings x, y, and z such that $s = xyz$ and

   (a) $xy^iz \in A$ for each $i \geq 0$,

   (b) $|y| > 0$,

   (c) $|xy| \leq p$.

   Since the first p symbols of s are all a's, the third property implies that x and y consist only of a's. So z will be the rest of the a's, followed by $c^{3p}b^{2p}$. The second property states that $|y| > 0$, so y has at least one a. More precisely, we can then say that

$$\begin{aligned} x &= a^j \text{ for some } j \geq 0, \\ y &= a^k \text{ for some } k \geq 1, \\ z &= a^m c^{3p} b^{2p} \text{ for some } m \geq 0. \end{aligned}$$

   Since $a^{2p}c^{3p}b^{2p} = s = xyz = a^j a^k a^m c^{3p} b^{2p} = a^{j+k+m} c^{3p} b^{2p}$, we must have that

$$j + k + m = 2p.$$

   The first property implies that $xy^2z \in A$, but

$$\begin{aligned} xy^2z &= a^j a^k a^k a^m c^{3p} b^{2p} \\ &= a^{2p+k} c^{3p} b^{2p} \end{aligned}$$

   since $j + k + m = 2p$. Hence, $xy^2z \notin A$ since $k \geq 1$, and we get a contradiction. Therefore, A is a nonregular language.