Midterm Exam 1
CS 341: Foundations of Computer Science II — **Fall 2012, day section**
Prof. Marvin K. Nakayama

Print family (or last) name: ____

Print given (or first) name: ____

I have read and understand all of the instructions below, and I will obey the Academic Honor Code.

____
Signature and Date

- This exam has 7 pages in total, numbered 1 to 7. Make sure your exam has all the pages.
- Note the number written on the upper right-hand corner of the first page. On the sign-up sheet being passed around, sign your name next to this number.
- This exam will be 1 hour and 25 minutes in length.
- This is a closed-book, closed-note exam.
- For all problems, follow these instructions:
  1. Give only your answers in the spaces provided. I will only grade what you put in the answer space, and I will take off points for any scratch work in the answer space. Use the scratch-work area or the backs of the sheets to work out your answers before filling in the answer space.
  2. For any proofs, be sure to provide a step-by-step argument, with justifications for every step.

Problem	1	2	3	4	5	Total
Points						

1. **[20 points]** For each of the following, circle TRUE if the statement is correct. Otherwise, circle FALSE

(a) TRUE FALSE — A language has a regular expression if and only if it has an NFA.

(b) TRUE FALSE — If we remove a finite set of strings from a nonregular language, then the result is a nonregular language.

(c) TRUE FALSE — If a language A has a PDA, then A is generated by a context-free grammar in Chomsky normal form.

(d) TRUE FALSE — If A is a context-free language and B is a language such that $B \subseteq A$, then B must be a context-free language.

(e) TRUE FALSE — The language $\{ 0^n 1^n \mid 0 \leq n \leq 1000 \}$ is regular.

(f) TRUE FALSE — If a language is context-free, then it must be regular.

(g) TRUE FALSE — If a language is regular, then it must be finite.

(h) TRUE FALSE — Nonregular languages are recognized by NFAs.

(i) TRUE FALSE — The class of context-free languages is closed under intersection.

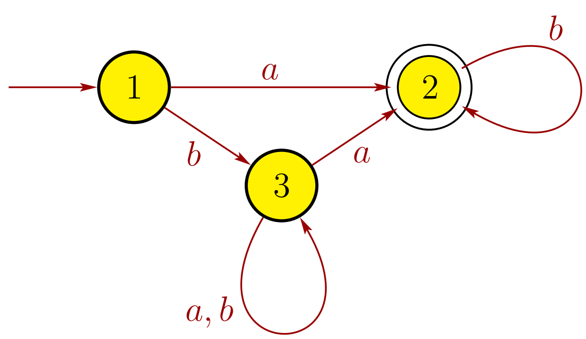
(j) TRUE FALSE — If a language A has a regular expression, then $\overline{A}$ must be a context-free language.

2. **[20 points]** Give short answers to each of the following parts. **Each answer should be at most a few sentences. Be sure to define any notation that you use.**

   (a) Let $\Sigma = \{0, 1\}$, and let A be the set of strings over Σ having an odd number of 0's. Give a regular expression for A.

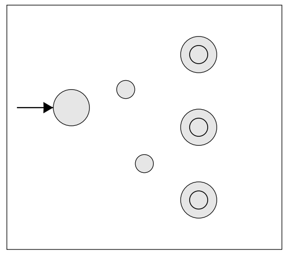
   (b) Give a regular expression for the language recognized by the NFA below.

   Answer: ______________________________

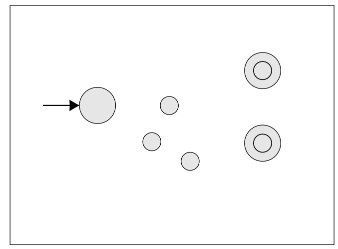


(c) Suppose that language A_1 is recognized by NFA N_1 below, and language A_2 is recognized by NFA N_2 below. Note that the transitions are not drawn in N_1 and N_2. Draw a picture of an NFA for $A_1 \circ A_2$.

N_1

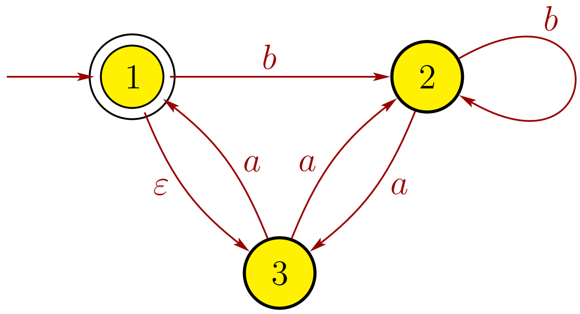


N_2



(d) Suppose that A_1 is a language defined by a CFG $G_1 = (V_1, \Sigma, R_1, S_1)$, and A_2 is a language defined by a CFG $G_2 = (V_2, \Sigma, R_2, S_2)$, where the alphabet Σ is the same for both languages and $V_1 \cap V_2 = \emptyset$. Let $A_3 = A_1 \cup A_2$. Give a CFG G_3 for A_3 in terms of G_1 and G_2. You do not have to prove the correctness of your CFG G_3, but do not give just an example.

3. **[20 points]** Let N be the following NFA with $\Sigma = \{a, b\}$, and let $C = L(N)$.



Give a DFA for C.

Scratch-work area

4. **[25 points]** Consider the alphabet $\Sigma = \{a, b\}$ and the language

$$L = \{\ b^{3n} a^{2n} \mid n \geq 0\ \}.$$

(a) Give a context-free grammar G for L. Be sure to specify G as a 4-tuple $G = (V, \Sigma, R, S)$.

(b) Give a PDA for L. You only need to draw the graph.

Scratch-work area

5. **[15 points]** Recall the pumping lemma for regular languages:

Theorem: If L is a regular language, then there exists a pumping length p where, if $s \in L$ with $|s| \geq p$, then there exists strings x, y, z such that $s = xyz$ and (i) $xy^iz \in L$ for each $i \geq 0$, (ii) $|y| \geq 1$, and (iii) $|xy| \leq p$.

Let $A = \{\, a^{2n}c^{3n}b^{2n} \mid n \geq 0 \,\}$. Is A a regular or nonregular language? If A is regular, give a regular expression for A. If A is not regular, prove that it is a nonregular language.

Circle one: **Regular Language** **Nonregular Language**