

Midterm Exam 1
CS 341: Foundations of Computer Science II — **Spring 2012, day section**
Prof. Marvin K. Nakayama

Print family (or last) name: _____

Print given (or first) name: _____

I have read and understand all of the instructions below, and I will obey the Academic Honor Code.

Signature and Date

- This exam has 9 pages in total, numbered 1 to 9. Make sure your exam has all the pages.
- Note the number written on the upper right-hand corner of the first page. On the sign-up sheet being passed around, sign your name next to this number.
- This exam will be 1 hour and 25 minutes in length.
- This is a closed-book, closed-note exam.
- For all problems, follow these instructions:
 1. Give only your answers in the spaces provided. I will only grade what you put in the answer space, and I will take off points for any scratch work in the answer space. Use the scratch-work area or the backs of the sheets to work out your answers before filling in the answer space.
 2. For any proofs, be sure to provide a step-by-step argument, with justifications for every step.

Problem	1	2	3	4	5	Total
Points						

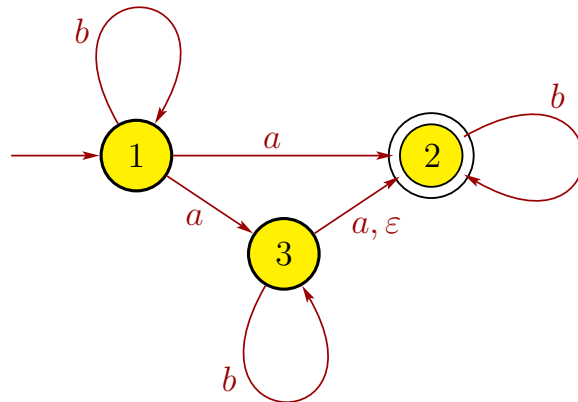
1. [20 points] For each of the following, circle TRUE if the statement is correct. Otherwise, circle FALSE

- (a) TRUE FALSE — The class of context-free languages is closed under union.
- (b) TRUE FALSE — If a finite number of strings is added to a regular language A , then the resulting language is regular.
- (c) TRUE FALSE — If a finite number of strings is added to a nonregular language A , then the resulting language is nonregular.
- (d) TRUE FALSE — If A is a regular language, then A is finite.
- (e) TRUE FALSE — If A and B are regular languages, then $(\overline{A} \cup B)^*$ is regular.
- (f) TRUE FALSE — The class of context-free languages is closed under intersection.
- (g) TRUE FALSE — If B is a context-free language and $A \subseteq B$, then A is context-free.
- (h) TRUE FALSE — The language $\{a^n b^n \mid n \geq 3\}$ is non-regular.
- (i) TRUE FALSE — If a language A has a regular expression, then A has a CFG in Chomsky normal form.
- (j) TRUE FALSE — If A is a non-context-free language, then A is also non-regular.

2. [20 points] Give short answers to each of the following parts. Be sure to define any notation that you use.

(a) Give a regular expression for the language recognized by the NFA below.

Answer: _____



(b) Suppose A is generated by a context-free grammar $G = (V, \Sigma, R, S)$. Give a context-free grammar G' for A^* in terms of G . You do not have to prove the correctness of your CFG G' , but do not just give an example.

- (c) Let $M_1 = (Q_1, \Sigma, \delta_1, q_1, F_1)$ be a DFA with language A_1 , and $M_2 = (Q_2, \Sigma, \delta_2, q_2, F_2)$ be a DFA with language A_2 . Consider the language $A = A_1 \cap A_2$. Give a DFA M_3 for A in terms of M_1 and M_2 . Your DFA M_3 must be completely general. Do not prove the correctness of your DFA M_3 , but do not just give an example.

- (d) Suppose that we are in the process of converting a CFG G with $\Sigma = \{0, 1\}$ into Chomsky normal form. We have already applied some steps in the process, and we currently have the following CFG:

$$\begin{aligned} S_0 &\rightarrow S \\ S &\rightarrow A1SA \mid A0 \mid \varepsilon \\ A &\rightarrow 0S0 \mid \varepsilon \end{aligned}$$

In the next step, we want to remove the ε -rule $A \rightarrow \varepsilon$. Give the CFG after carrying out just this one step.

3. [25 points] Define L to be the set of strings that represent numbers in a modified version of Java. The goal in this problem is to define a regular expression and an NFA for L . To precisely define L , let $\Sigma = \Sigma_1 \cup \Sigma_2 \cup \{.\}$, where $\Sigma_1 = \{0, 1, 2, \dots, 9\}$ is the set of *digits* and $\Sigma_2 = \{+, -\}$ is the set of *signs*. Then $L = L_1 \cup L_2$, where

- L_1 is the set of all strings that are decimal integer numbers. Specifically, L_1 consists of strings that start with an optional sign, followed by one or more digits. Examples of strings in L_1 are “02”, “+9”, and “-241”.
- L_2 is the set of all strings that are floating-point numbers that are not in exponential notation. Specifically, L_2 consists of strings that start with an optional sign, followed by zero or more digits, followed by a decimal point, and end with zero or more digits, where there must be at least one digit in the string. Examples of strings in L_2 are “13.231”, “-28.” and “.124”. All strings in L_2 have exactly one decimal point.

Assume that there is no limit on the number of digits in a string in L . Also, we do not allow exponential notation, nor do we allow for the suffixes L, l, F, f, D, d , at the end of numbers to denote types (long integers, floats, and doubles); these symbols are not in Σ anyways.

(a) Give a regular expression for L_1 .

(b) Give an NFA for L_1 over the alphabet Σ .

(c) Give a DFA for L_1 over the alphabet Σ . Your DFA must include all transitions.

(d) Give a regular expression for L_2 .

(e) Give an NFA for L_2 over the alphabet Σ .

(f) Give a regular expression for L .

(g) Give an NFA for L over the alphabet Σ .

4. [20 points] Consider the alphabet $\Sigma = \{a, b, c\}$ and the language

$$L = \{ b^i a^j c^k \mid i, j, k \geq 0 \text{ and } i + j = k \}.$$

(a) Give a context-free grammar G for L . Be sure to specify G as a 4-tuple $G = (V, \Sigma, R, S)$.

(b) Give a PDA for L . You only need to draw the graph.

Scratch-work area

5. [15 points] Recall the pumping lemma for regular languages:

Theorem: If L is a regular language, then there exists a pumping length p where, if $s \in L$ with $|s| \geq p$, then there exists strings x, y, z such that $s = xyz$ and (i) $xy^iz \in L$ for each $i \geq 0$, (ii) $|y| \geq 1$, and (iii) $|xy| \leq p$.

Let $A = \{ww \mid w \in \{a, b\}^*\}$. Is A a regular or nonregular language? If A is regular, give a regular expression for A . If A is not regular, prove that it is a nonregular language.

Circle one:

Regular Language

Nonregular Language